Norwegian University of Science and Technology
Department of Mathematical Sciences

MA8105
Nonlinear PDEs and Sobolev spaces
Spring 2019

Exercise set 8

$\boxed{1}$ Verify the claims in Remark 4.8 p 64 in Holden.

$\boxed{2}$ Give the details to the proof of Theorem 4.9 p 64 in Holden.

Hint: 1. Show $\int f_n\phi \to \int f\phi$ for all simple functions ϕ. 2. Use the fact that simple functions are dense in L^q for $q \in [1, \infty)$ to conclude.

$\boxed{3}$ (Holden Ex 9 p 93) Show that $f_n = n\chi_{(0,\frac{1}{n})}$ is not uniformly integrable on $X = (0, 1)$ in two ways:

(a) Using directly the definition, and

(b) Showing that the sequence converge to δ_0 in distributions – and hence has no convergence subsequence in $L^1(0, 1)$ – and then conclude by Dunford-Pettis.

$\boxed{4}$ (See also Holden Ex 10 p 93) Let $f_n(x) = g(x+n)$ for some $0 \not\equiv g \in L^p(\mathbb{R})$, $p \in (1, \infty)$. Show that $f_n \to 0$ weakly in $L^p(\mathbb{R})$.

Hint: Show convergence of means on intervals.

$\boxed{5}$ (Holden Ex 11 p 93) Show the $p = 1$ case in Example 4.25 (i) p 72 in Holden.