Norwegian University of Science
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Department of Mathematical Sciences

MA8105
Nonlinear PDEs and Sobolev spaces
Spring 2019

Exercise set 9

1 Show that $f_n \to f$ in L^1 implies $f_n \to f$ in $\mathcal{M}$.

Explain how to interpret the latter convergence.

2 Prove that Thm 67 (compactness in $(C_c)'$) in my notes (see 'Lectures' on the webpage) implies Thm 65 (compactness in $(C_0)'$)).

Hint: This is easy, approximate C_0 functions by C_c functions.

This material is partly taken from Folland: *Real Analysis* chp 7, partly from Holden.

3 Prove that the following two statements are equivalent:

(a) $f_n \to f$ in $W^{1,p}(K)$, $\forall K \subset \Omega$ compact.

(b) $f_n\phi \to f\phi$ in $W^{1,p}(\Omega)$, $\forall \phi \in C_c^\infty(\Omega)$.

4 Prove that $W^{m,p}(\Omega)$ is a Banach space.

Hint: Show 1) normed space and 2) completeness. Use the completeness of L^p.

5 Let $\phi \in C_b^m(\Omega)$ (all derivatives of order less than m are bounded, continuous) and $f \in W^{m,p}(\Omega)$, show that

$$\|\phi f\|_{m,p} \leq C\|\phi\|_{m,\infty}\|f\|_{m,p}.$$

Hint: Product rule, induction.