MA8109
Stochastic Processes in
Systems Theory
Autumn 2013

Norwegian University of Science
and Technology
Department of Mathematical
Sciences

Exercise set 4

$\boxed{1}$ (Exam 2003, Problem 3b) Solve the stochastic differential equation

(1) $$dX_t = \frac{1}{t}X_t dt + t\, dB_t, \ t \geq 1, \ X_1 = 1.$$

Hint: Use an integrating factor (for the dt-part of the equation).

$\boxed{2}$ Øksendal Exercise 5:5.4

$\boxed{3^*}$ Øksendal Exercise 5:5.5

$\boxed{4}$ Øksendal Exercise 5:5.6

$\boxed{5}$ Øksendal Exercise 5:5.7

$\boxed{6^*}$ Øksendal Exercise 5:5.9

$\boxed{7^*}$ Øksendal Exercise 5:5.10

$\boxed{8}$ Øksendal Exercise 5:5.11

Hint: Show that $X_t \to b$ in $L^2(\Omega)$ as $t \to 1^-$ (direct argument and Ito isometry) and use the Doob martingale inequality and Borel-Cantelli. See the hint at the end of Øksendal for more details.

More hints at the end of Øksendal.