Norwegian University of Science and Technology
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MA8404 Numerical solution of time dependent differential equations
Autumn 2017

Exercise set 2

1 GNI II.6, Problem 1-4 and 6a, (no implemention required).

2 In GNI IV, Example 1.3, conservation of total linear and angular momentum is discussed. Prove that in the case of a Kepler problem, with the Hamiltonian

$$H(p,q) = \frac{1}{2}p^T M^{-1} p - \frac{1}{\|q\|_2}$$

also the vector

$$A = p \times L - M\frac{q}{\|q\|_2}$$

is conserved. Here, $L = q \times p$ is the angular momentum, $q, p \in \mathbb{R}^3$ and $M \in \mathbb{R}^+$ is the mass of the body.

Hint: Use the identity $x \times (y \times z) = (x^T z)y - (x^T y)z$.

3 For a given Hamiltonian

$$H(p,q) = \frac{1}{2}p^2 + V(q)$$

the Hamiltons equations becomes

$$q' = p, \qquad p' = -V'(q)$$

Use the Morse potential $V(q) = (1 - e^{-q})^2$. Solve this problem by the explicit Euler method, RK4 (the classical 4th order method) and Störmer-Verlet's method. Plot the solution in the $p-q$ plane. Examine the energy conservation H for the different methods. Experiment with different stepsizes. As initial values, choose e.g. $q_0 = 1$, $p_0 = 1$ and integrate from 0 to 20 (for example). You may very well also experiment with different initial values.

4 Set up an extrapolation scheme based on the midpoint rule and the *Romberg sequence*

$$1, 2, 4, 8, 16, \ldots$$

The numerical solution after a given number of extrapolation steps can be written as a Runge-Kutta method. Write down the Butcher tableau for the 4th and 6th order method. Will these methods preserve the geometric properties (symmetry, conservation of quadratic invariants) of the original method?