

FASIT, øvingsoppgaver

① a) $y = \sqrt{x^3} = (x^3)^{1/2} = x^{3/2}$

$$\frac{dy}{dx} = \frac{3}{2} x^{1/2}$$

b) $\frac{dy}{dx} = \frac{5}{2} (x^2 + 4x)^{3/2} (2x + 4) = 5(x+2)(x^2 + 4x)^{3/2}$

c) $\frac{dy}{dx} = \frac{(4x^3 + 2x)(x^2 + x + 1) - (x^4 + x^2)(2x + 1)}{(x^2 + x + 1)} \quad (\text{forkort})$

d) $\frac{dy}{dx} = 9 \cdot (-1) x^{-2} = -9x^{-2}$

e) $y = \sqrt{\frac{1}{5x^6}} = \frac{1}{\sqrt{5x^6}} = \frac{1}{\sqrt{5} \cdot \sqrt{x^6}} = \frac{1}{\sqrt{5} (x^6)^{1/2}} = \frac{1}{\sqrt{5} x^3} = \frac{1}{\sqrt{5}} x^{-3}$

$$\frac{dy}{dx} = -\frac{3}{\sqrt{5}} x^{-4}$$

f) $\frac{dy}{dx} = \frac{1}{4} (12x + \frac{1}{2} x^{-1/2})$

g) $\frac{dy}{dx} = \frac{1}{2\sqrt{1+\sin\sqrt{x}}} \cdot (\cos\sqrt{x}) \cdot \frac{1}{2\sqrt{x}} = \frac{\cos\sqrt{x}}{4\sqrt{x}\sqrt{1+\sin\sqrt{x}}}$

h) $\frac{dy}{dx} = \frac{6x^5 + 4x^3}{2\sqrt{x^6 + x^4}} = \frac{2x^3(3x^2 + 2)}{2\sqrt{x^4(x^2 + 1)}} = \frac{x^3(3x^2 + 2)}{x^2\sqrt{x^2 + 1}}$
$$= \frac{x(3x^2 + 2)}{\sqrt{x^2 + 1}}$$

② $f'(x) = 1 - \frac{1}{x^2} = 0 \Rightarrow x = \pm 1 \quad f(1) = 2, f(-1) = -2$

$f''(x) = \frac{2}{x^3}, \quad f''(1) = 2 > 0 \Rightarrow (1, 2) \text{ min. pkt.}$

$f''(-1) = -2 < 0 \Rightarrow (-1, -2) \text{ max. pkt.}$

④ Skjøningssetningen viser at det fins ett nullpunkt (minst ett). Hvis vi i tillegg kan vise at f er strengt voksende/avtagende får vi at det fins maks ett.

a) $-1,61$ b) $-0,51$

⑤ $3,141592654$