$$\sum_{j=1}^{n-1} j^3 \leq \frac{n^4}{4} \leq \sum_{j=1}^{n} j^3 \qquad ; \quad n \geq 2.$$

i) Ser på $\displaystyle\sum_{j=1}^{n-1} j^3 \leq \frac{n^4}{4}$ :

1) $n = 2$ : $\displaystyle V.S = \sum_{j=1}^{1} j^3 = 1^3 = 1.$

$\displaystyle H.S = \frac{2^4}{4} = \frac{16}{4} = 4.$

$\Big\}$ $V.S \leq H.S.$ ok.

2) Antar $\displaystyle\sum_{j=1}^{k-1} j^3 \leq \frac{k^4}{4}$.

3) (Skal vise : $\displaystyle\sum_{j=1}^{k} j^3 \leq \frac{(k+1)^4}{4}$.)

$$\sum_{j=1}^{k} j^3 = \sum_{j=1}^{k-1} j^3 + k^3 \leq \frac{k^4}{4} + k^3$$

$$= \frac{k^4 + 4k^3}{4}$$

$$\leq \frac{k^4 + 4k^3 + \overbrace{6k^2 + 4k + 1}^{\text{legger til for å få } (k+1)^4 \ldots}}{4}$$

$$= \frac{(k+1)^4}{4}.$$

ii) Ser på $\frac{n^4}{4} \leq \sum_{j=1}^{n} j^3$ .

1) $n=2$ : $V.S = \frac{2^4}{4} = 4$

$H.S = \sum_{j=1}^{2} j^3 = 1^3 + 2^3 = 9$ $\Bigg\}$ $V.S \leq H.S.$ ok.

2) Antar $\frac{k^4}{4} \leq \sum_{j=1}^{k} j^3$ .

3) (Skal vise $\frac{(k+1)^4}{4} \leq \sum_{j=1}^{k+1} j^3$ )

$$\frac{(k+1)^4}{4} = \frac{k^4 + 4k^3 + 6k^2 + 4k + 1}{4}$$

$$= \frac{k^4}{4} + \frac{4k^3 + 6k^2 + 4k + 1}{4} \leq \sum_{j=1}^{k} j^3 + k^3 + \frac{3}{2}k^2 + k + \frac{1}{4}$$

$$\leq \sum_{j=1}^{k} j^3 + k^3 + 3k^2 + 3k + 1$$

$$= \sum_{j=1}^{k} j^3 + (k+1)^3 = \sum_{j=1}^{k+1} j^3 .$$

$\blacksquare$