TMA4100 Øving 2

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Exercise 3.7.99

If $y = A\sin(\ln x) + B\cos(\ln x)$, where A and B are constants, show that

$$x^2y'' + xy' + y = 0.$$

Exercise 3.8.10

Use reference triangles like that in Example 1 to find the angles

a) $\csc^{-1}(-\sqrt{2})$

b) $\csc^{-1}(\frac{2}{\sqrt{3}})$

c) $\csc^{-1}(-2)$

Exercise 3.9.13

A 13-ft ladder is leaning against a house when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving at a rate of 5 ft/sec.

a) How fast is the top of the ladder sliding down the wall then?

b) At what rate is the area of the triangle formed by the ladder, wall, and ground changing then?

c) At what rate is the angle θ between the ladder and the ground changing then?

Exercise 3.9.19

Water is flowing at a rate of 6 m^3/min from a reservoir shaped like a hemispherical bowl of radius 13 m. Answer the following questions, given that the volume of water in a hemispherical bowl of radius R is $V = (\pi/3)y^2(3R - y)$ when the water is y meters deep.

a) At what rate is the water level changing when the water is 8 m deep?

b) What is the radius r of the water's surface when the water is y m deep?

c) At what rate is the radius r changing when the water is 8 m deep?

Exercise 3.10.54

A surveyor, standing 30 ft from the base of a building, measures the angle of elevation to the top of the building to be 75°. How accurately must the angle be measured for the percentage error in estimating the height of the building to be less than 4%?

Exercise 3.11.44

Derive the formula

$$\sinh^{-1} x = \ln\left(x + \sqrt{1 + x^2}\right), \quad -\infty < x < \infty.$$

Explain in your derivation why the plus sign is used with the square root instead of the minus sign.

Exercise 4.1.23

Find the absolute maximum and minimum value of the function

$$g(x) = \sqrt{4 - x^2}, \quad -2 \le x \le 1.$$

Graph the function. Identify the points on the graph where the absolute extrema occur, and include their coordinates.

Exercise 4.1.70

Consider the cubic function

$$f(x) = ax^3 + bx^2 + cx + d.$$

a) Show that f can have 0, 1, or 2 critical points. Give examples and graphs to support your argument.

b) How many local extreme values can f have?

Exercise 4.2.5-8

Which of the following functions satisfy the hypotheses of the Mean Value Theorem on the given interval, and which do not? Give reasons for your answers.

a) $f(x) = x^{2/3}, \quad [-1, 8]$

b) $f(x) = x^{4/5}, \quad [0, 1]$

c) $f(x) = \sqrt{x(1 - x)}, \quad [0, 1]$

d)

$$f(x) = \begin{cases} \dfrac{\sin x}{x}, & -\pi \le x < 0 \\ 0, & x = 0 \end{cases}$$

Exercise 4.2.20

Show that the function

$$r(\theta) = 2\theta - \cos^2\theta + \sqrt{2}$$

has exactly one zero on the interval $(-\infty, \infty)$.

Exercise 4.2.58

Show that for any numbers a and b, the sine inequality

$$|\sin b - \sin a| \leq |b - a|$$

is true.

Siste oppgave

Vis at grafen til ligningen

$$x^3 + y^3 = xy - 1$$

ikke har horisontal tangent ($dy/dx = 0$) i noen punkter.