# TMA4100 Øving 3

September 14, 2011

### Exercise 3.11.49

Imagine a cable, like a telephone line or TV cable, strung from one support to another and hanging freely. The cable's weight per unit length is a constant $w$ and the horizontal tension at its lowest point is a vector of length $H$. If we choose a coordinate system for the plane of the cable in which the $x$-axis is horizontal, the force of gravity is straight down, the positive $y$-axis points straight up, and the lowest point of the cable lies at the point $y = H/w$ on the $y$-axis, then it can be shown that the cable lies along the graph of the hyperbolic cosine

$$y = \frac{H}{w} \cosh \frac{w}{H} x.$$

a) Let $P(x, y)$ denote an arbitrary point on the cable. The tension at point $P$ is a vector of length $T$. Show that the cable's slope at $P$ is

$$\tan \phi = \frac{dy}{dx} = \sinh \frac{w}{H} x.$$

b) Using the result from part a) and the fact that the horizontal tension at $P$ must equal $H$ (the cable is not moving), show that $T = wy$. Hence, the magnitude of the tension at $P(x, y)$ is exactly equal to the weight of $y$ units of cable.

Merknad: Oppgaveteksten har to figurer som er essensielle for å forstå oppgaven. Finn ei bok å se i!

### Exercise 4.3.18

Let

$$g(x) = x^4 - 4x^3 + 4x^2.$$

a) Find the intervals on which $g$ is increasing and decreasing.

b) Identify the function's local extreme values, if any, and say where they are taken on.

c) Which, if any, of the extreme values are absolute?

### Exercise 4.3.46

Show that the function

$$h(\theta) = 5\sin\frac{\theta}{2}, \quad 0 \le \theta \le \pi,$$

has local extreme values at $\theta = 0$ and $\theta = 2\pi$, and say which kind of local extremes these are.

### Exercise 4.3.58

a) Prove that $e^x \ge 1 + x$ if $x \ge 0$.

b) Use the result in part a) to show that

$$e^x \ge 1 + x + \frac{1}{2}x^2.$$

### Exercises 4.4.10

Use the steps of the graphing procedure on page 264 to graph the equation

$$y = 6 - 2x - x^2.$$

### Exercises 4.4.22

Use the steps of the graphing procedure on page 264 to graph the equation

$$y = x - \sin x, \quad 0 \le x \le 2\pi.$$

### Exercises 4.4.26

Use the steps of the graphing procedure on page 264 to graph the equation

$$y = x^{2/3}\left(\frac{5}{2} - x\right).$$

### Exercises 4.4.30

Use the steps of the graphing procedure on page 264 to graph the equation

$$y = \frac{x^3}{3x^2 + 1}.$$

### Exercise 4.5.24

Figure 1 shows a cross-section of a 20 ft trough. The sides of the trough, $A$, $B$ and $C$, all have length 1 ft. Only the angle $\theta$ can be varied. What value of $\theta$ will maximize the trough's volume?

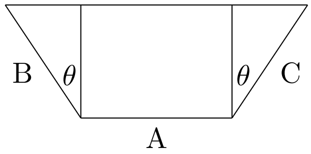


Figure 1: Cross-section of 20 ft long trough

### Exercise 4.5.40

Fermat's principle in optics states that light always travels from one point to another along a path that minimizes the travel time. Light from a source $A$ is reflected by a plane mirror to a reciever at point $B$ (sketch this!). Show that for the light to obey Fermat's principle, the angle of incidence must equal the angle of reflection, both measured from the line normal to the reflecting surface. (This result can also be derived without calculus. There is a purely geometric argument which you may prefer.)

### Exercise 4.5.48

Suppose that

$$c(x) = x^3 - 20x^2 + 20000x$$

is the cost of manufacturing $x$ items. Find a production level that will minimize the average cost of making $x$ items.

### Exercise 4.6.2

Use l'Hôpital's Rule to evaluate the limit

$$\lim_{x \to 0} \frac{\sin 5x}{x}.$$

Then evaluate the same limit using a method studied in Chapter 2.

### Exercise 4.6.40

Use l'Hôpital's Rule to evaluate the limit

$$\lim_{x \to 0^+} \left( \frac{3x+1}{x} - \frac{1}{\sin x} \right).$$

### Exercise 4.6.58

L'Hôpital's Rule does not help with the limit

$$\lim_{x \to 0^+} \frac{\sqrt{x}}{\sqrt{\sin x}}.$$

Try it - you just keep on cycling. Find the limit some other way.

**Exercise 4.6.70**

Given that $x > 0$, find the maximum value, if any, of

a) $x^{1/x}$

b) $x^{1/x^2}$

c) $x^{1/x^n}$ ($n$ a positive integer)

d) Show that $\lim_{x\to\infty} x^{1/x^n} = 1$ for every positive integer $n$.