TMA4100 Øving 1

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Exercise 1.5.59

Find a formula for the inverse function f^{-1} and verify that $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$.

(a)

$$f(x) = \frac{100}{1 + 2^{-x}}$$

(b)

$$f(x) = \frac{50}{1 + 1.1^{-x}}$$

Exercise 2.1.3

Find the average rate of change of the function $h(t) = \cot t$ over the intervals

(a)

$$[\pi/4, 3\pi/4] \text{ , and}$$

(b)

$$[\pi/6, \pi/2] \text{ .}$$

Exercise 2.3.20

We have

$$f(x) = \sqrt{x - 7}, \quad L = 4, \quad x_0 = 23, \quad \epsilon = 1 \text{ .}$$

First, find an open interval about x_0 on which the inequality $|f(x) - L| < \epsilon$ holds. Then give a value for $\delta > 0$ such that for all x satisfying $0 < |x - x_0| < \delta$ the inequality $|f(x) - L| < \epsilon$ holds.

Exercise 2.3.53

Show by example that the following statement is wrong: *The number L is the limit of $f(x)$ as x approaches x_0 if $f(x)$ gets closer to L as x approaches x_0.*

Explain why the function in your example does not have the given value of L as a limit as $x \to x_0$.

Exercise 2.5.35

Graph the rational function

$$y = \frac{x^2 - 4}{x - 1} \ .$$

Include the graphs and equations of its asymptotes.

Exercise 2.6.39

For what value of a is

$$f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$$

continuous at every x?

Exercise 2.6.46

Explain why the equation $\cos x = x$ has at least one solution.

Exercise 2.7.34

Does the graph

$$g(x) = \begin{cases} x \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

have a tangent at the origin? Give reasons for your answer.

Exercise 3.2.53

The curve $y = ax^2 + bx + c$ passes through the point $(1, 2)$ and is tangent to the line $y = x$ at the origin. Find a, b and c.

Exercise 3.3.13

Had Galileo dropped a cannonball from the Tower of Pisa, 179 ft above the ground, the ball's height above the ground t sec into the fall would have been

$$s = 179 - 16t^2 \ .$$

a) What would have been the ball's velocity, speed and acceleration at time t?

b) About how long would it have taken the ball to hit the ground?

c) What would have been the ball's velocity at the moment of impact?

Exercise 3.4.25

Find y'' if

1. $y = \csc x$.
2. $y = \sec x$.

Exercise 3.5.97

Find a parametrization for the lower half of the parabola $x - 1 = y^2$.

Exercise 3.6.2

Use implicit differentiation to find dy/dx when

$$x^3 + y^3 = 18xy \ .$$

Exercise 3.6.32

Verify that the point $(-2, 1)$ is on the curve

$$y^2 - 2x - 4y - 1 = 0 \ ,$$

and find the lines that are **(a)** tangent and **(b)** normal to the curve at the given point.

Exercise 3.7.9

Suppose that the differentiable function $y = f(x)$ has an inverse and that the graph of f passes through the point $(2, 4)$ and has a slope of $1/3$ there. Find the value of df^{-1}/dx at $x = 4$.

Exercise 3.8.13

Find the limit

$$\lim_{x \to 1^-} \sin^{-1} x \ .$$