TMA4100 Øving 4

September 25, 2012

Exercise 4.7.3

Use Newton's method to estimate the two zeros of the function $f(x) = x^4 + x - 3$. Start with $x_0 = -1$ for the left-hand zero and with $x_0 = 1$ for the zero on the right. Then, in each case, find x_2.

Exercise 4.7.8

Estimating pi You plan to estimate $\pi/2$ to five desimal places by using Newton's method to solve the equation $\cos x = 0$. Does it matter what your starting value is? Give reasons for your answer.

Exercise 4.7.13

Intersecting curves The curve $y = \tan x$ crosses the line $y = 2x$ between $x = 0$ and $x = \pi/2$. Use Newton's method to find where.

Exercise 4.7.25

Curves that are nearly flat at the root Some curves are so flat that, in practice, Newton's method stops too far from the root to give a useful estimate. Try Newton's method on $f(x) = (x-1)^{40}$ with a starting value of $x_0 = 2$ to see how close your machine comes to the root $x = 1$.

Exercise 4.8.24

Find an antiderivative for each function. Check your answers by differentiation.

a. $x - \left(\frac{1}{2}\right)^x$

b. $x^2 + 2^x$

c. $\pi^x - x^{-1}$

Exercise 4.8.76

Verify the formula by differentiation.

$$\int \frac{1}{(x+1)^2}\mathrm{d}x = \frac{x}{x+1} + C$$

Exercise 4.8.85

Right or wrong? Say which for each formula and give a brief reason for each answer.

a. $\displaystyle\int (2x+1)^2\mathrm{d}x = \frac{(2x+1)^3}{3} + C$

b. $\displaystyle\int 3(2x+1)^2\mathrm{d}x = (2x+1)^3 + C$

c. $\displaystyle\int 6(2x+1)^2\mathrm{d}x = (2x+1)^3 + C$

Exercise 4.8.119

Stopping a car in time You are driving along a highway at a steady 60 mph (88 ft/sec) when you see an accident ahead and slam on the brakes. What constant deceleration is required to stop your car in 242 ft? To find out, carry out the following steps.

1. Solve the initial value problem

$$\begin{aligned}&\text{Differential equation:} && \frac{\mathrm{d}^2 s}{\mathrm{d}t^2} = -k \quad \text{(k constant)}\\ &\text{Initial conditions:} && \frac{\mathrm{d}s}{\mathrm{d}t} = 88 \text{ and } s = 0 \text{ when } t = 0.\end{aligned}$$

   Note: This measures the time and distance from when the brakes are applied.

2. Find the value of t that makes $\mathrm{d}s/\mathrm{d}t = 0$. (The answer will involve k.)

3. Find the value of k that makes $s = 242$ for the value of t you found in Step 2.

Exercise 5.1.7

Estimate the area under the graph of the function

$$f(x) = \frac{1}{x} \text{ between } x = 1 \text{ and } x = 5,$$

using rectangles whose height is given by the value of the function at the midpoint of the rectangle's base (*the midpoint rule*). Use first two and then four rectangles.

Exercise 5.1.19

Water pollution Oil is leaking out of a tanker damaged at sea. The damage to the tanker is worsening as evidenced by the increased leakage each hour, recorded in the following table.

Time (h)	0	1	2	3	4	5	6	7	8
Leakage (gal/h)	50	70	97	136	190	265	369	516	720

a. Give an upper and a lower estimate of the total quantity of oil that has escaped after 5 hours.

b. Repeat part (a) for the quantity of oil that has escaped after 8 hours.

c. The tanker continues to leak 720 gal/h after the first 8 hours. If the tanker originally contained 25,000 gal of oil, approximately how many more hours will elapse in the worst case before all the oil has spilled? In the best case?

Exercise 5.2.14

Express the sum

$$2 + 4 + 6 + 8 + 10$$

in sigma notation. The form of your answer will depend on your choice of the lower limit of summation.

Exercise 5.2.18d

Suppose that $\sum_{k=1}^{n} b_k = 1$. Find the value of

$$\sum_{k=1}^{n} (b_k - 1)$$

Exercise 5.2.39

For the function below find a formula for the upper sum obtained by dividing the interval $[a, b]$ into n equal subintervals. Then take the limit of these sums as $n \to \infty$ to calculate the area under the curve over $[a, b]$.

$$f(x) = x + x^2 \text{ over the interval } [0, 1].$$

Exercise 5.3.6

Express the limit

$$\lim_{||P||\to 0} \sum_{k=1}^{n} \sqrt{4 - c_k^2}\, \Delta x_k, \text{ where } P \text{ is a partition of } [0,1]$$

as a definite integral.

Exercise 5.3.18

Graph the integrand and use areas to evaluate the integral

$$\int_{-4}^{0} \sqrt{16 - x^2}\, \mathrm{d}x$$

Exercise 5.3.64

What values of a and b minimize the value of

$$\int_{a}^{b} (x^4 - 2x^2)\, \mathrm{d}x\,?$$

Exercise 5.3.68

Show that the value of $\int_0^1 \sqrt{x+8}\, \mathrm{d}x$ lies between $2\sqrt{2} \approx 2.8$ and 3.

Oppgave 19 fra eksamensoppgavesamlingen

Vis ved induksjon at

$$(\cos u)(\cos 2u)(\cos 4u)(\cos 8u) \cdots [\cos(2^{n-1}u)] = \frac{\sin(2^n u)}{2^n \sin u}$$

for alle hele tall $n \geq 1$.