

Key terms and concepts for sections 11.3–11.5

February 6, 2013

- The *arc length* (norsk: *buelengde*) of a smooth curve $\mathbf{r} : [a, b] \rightarrow \mathbb{R}^3$ is

$$\int_a^b |\mathbf{r}'(t)| dt.$$

- The *unit tangent vector* (*enhetstangentvektor*) of a smooth curve \mathbf{r} is

$$\mathbf{T}(t) = \frac{\mathbf{v}(t)}{|\mathbf{v}(t)|}$$

- The *curvature* (*krumning*) of a smooth curve with unit tangent vector \mathbf{T} is

$$\kappa = \left| \frac{d\mathbf{T}}{ds} \right|.$$

– By the chain rule

$$\kappa = \frac{1}{|\mathbf{v}|} \left| \frac{d\mathbf{T}}{dt} \right|.$$

– Computationally practical formula:

$$\kappa = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3}.$$

- The *principal unit normal vector* (*hovedenhetsnormalvektor*) of a smooth curve with tangent vector \mathbf{T} is

$$\mathbf{N} = \frac{1}{\kappa} \frac{d\mathbf{T}}{ds}.$$

– By the chain rule

$$\mathbf{N} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}.$$

- The *osculating circle* (*smygsirkel*) at a point has radius equal to the *radius of curvature* (*krumningsradius*) $\rho = 1/\kappa$, center in the *center of curvature*, and touches the curve at said point.

- The *Frenet frame* (*Frenet-rammen*) of a smooth curve is spanned by \mathbf{T} , \mathbf{N} and $\mathbf{B} = \mathbf{T} \times \mathbf{N}$.
- The *torsion* (*torsjon*) of a smooth curve is

$$\tau = -\frac{d\mathbf{B}}{ds} \cdot \mathbf{N}.$$