Løsningsforslag til eksamen i TMA4105 Matematikk 2, 22. mai 2013

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with(Student[VectorCalculus]):
with(Student[MultivariateCalculus]):
with(plots):
```

Oppgave 1

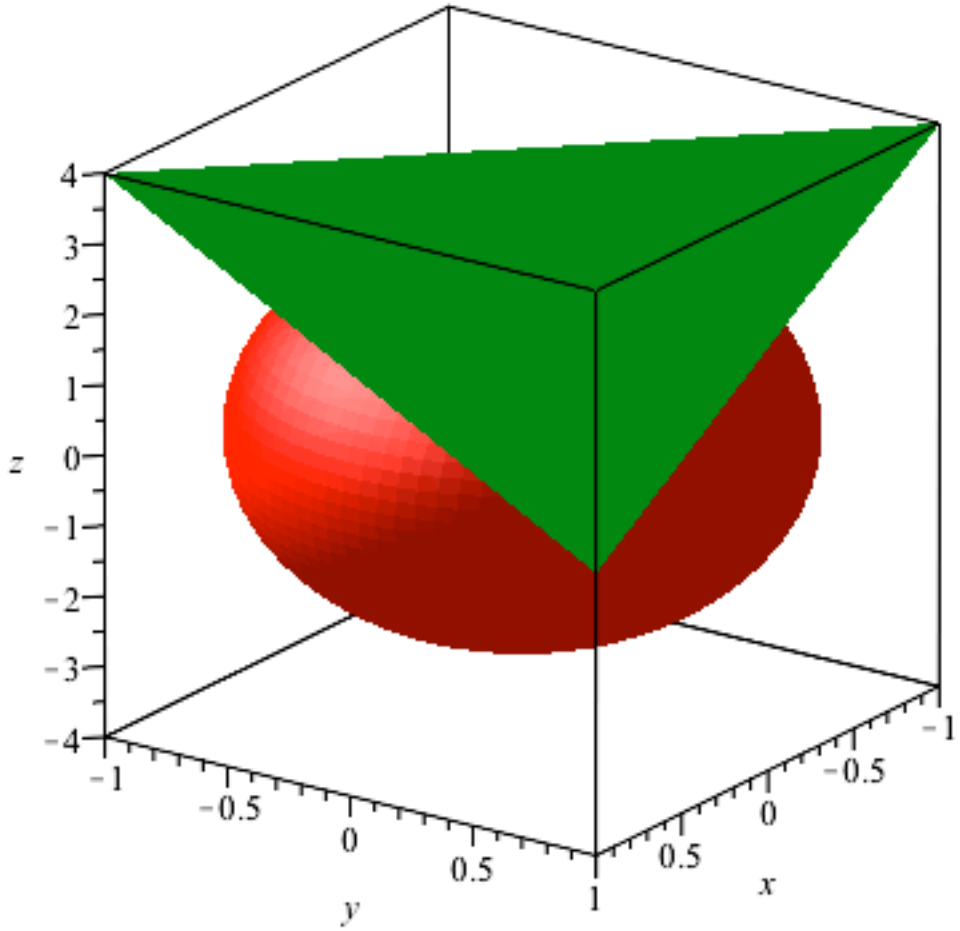
```
F := (x, y, z) -> x^2 + y^2 + z^2/8:
Del(F(x, y, z), [x, y, z] = [1/2, 1/2, 2]);
```

$$\begin{bmatrix} 1 \\ 1 \\ \frac{1}{2} \end{bmatrix} \tag{1}$$

```
<1, 1, 1/2> . <x - 1/2, y - 1/2, z - 2> = 0;
```

$$x - 2 + y + \frac{1}{2} z = 0 \tag{2}$$

```
display(implicitplot3d(x^2 + y^2 + z^2/8 = 1, x = -1..1, y = -1..1, z = -sqrt(8)..sqrt(8), color = "Red",
    numpoints = 20000, style = surface), implicitplot3d(%2, x = -1..1, y = -1..1, z = -4..4, color
    = "Green", numpoints = 20000, style = surface), axes = boxed, labels = [x, y, z], orientation = [35,
    70]);
```



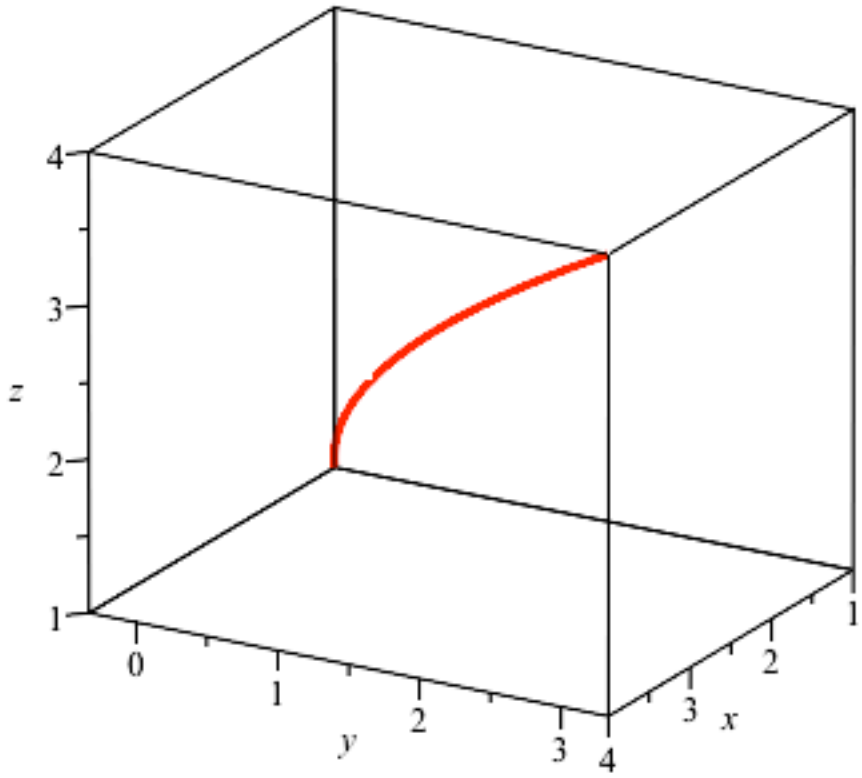
Oppgave 2

$r := t \to \left\langle t, \mathrm{sqrt}(t) \cdot \left(\frac{2}{3} \cdot t - 1 \right), t \right\rangle :$

$ArcLength(r(t), t = 1..4);$

$$\frac{17}{3} \qquad \textbf{(3)}$$

$spacecurve\left(\left\langle t, \mathrm{sqrt}(t) \cdot \left(\frac{2}{3} \cdot t - 1 \right), t \right\rangle, t = 1..4, thickness = 3, color = \text{"Red"}, axes = boxed, scaling = constrained, labels = [x, y, z], orientation = [30, 70] \right);$



Oppgave 3a

$f := (x, y) \to \frac{x^2 - y^2}{x - y}$:

$limit(f(x, y), \{x = a, y = a\})$;

$$2\,a \qquad \textbf{(4)}$$

Oppgave 3b

$DirectionalDerivative(x + y, [x, y] = [0, 0], [v1, v2])$;

$$\frac{v1}{\sqrt{v1^2 + v2^2}} + \frac{v2}{\sqrt{v1^2 + v2^2}} \qquad \textbf{(5)}$$

Oppgave 4

$f := (x, y, z) \to x^2 + y^2 + z^2$:

$g := (x, y, z) \to 2 \cdot x + 2 \cdot y + z - 4$:

$$LagrangeMultipliers(f(x,y,z),\ [g(x,y,z)],\ [x,y,z]);$$

$$\left[\frac{8}{9},\ \frac{8}{9},\ \frac{4}{9}\right] \tag{6}$$

$$\mathrm{sqrt}\left(\left(\frac{8}{9}\right)^2+\left(\frac{8}{9}\right)^2+\left(\frac{4}{9}\right)^2\right);$$

$$\frac{4}{3} \tag{7}$$

Oppgave 5

$$f := (x,y)\to 2\cdot x+2\cdot y:$$

$$MultiInt\left(\mathrm{sqrt}\left(1+(diff(f(x,y),x))^2+(diff(f(x,y),y))^2\right),\ x=y^2\ ..\mathrm{sqrt}(y),\ y=0\ ..1,\ output=steps\right);$$

$$\int_0^1\int_{y^2}^{\sqrt{y}} 3\ dx\ dy$$

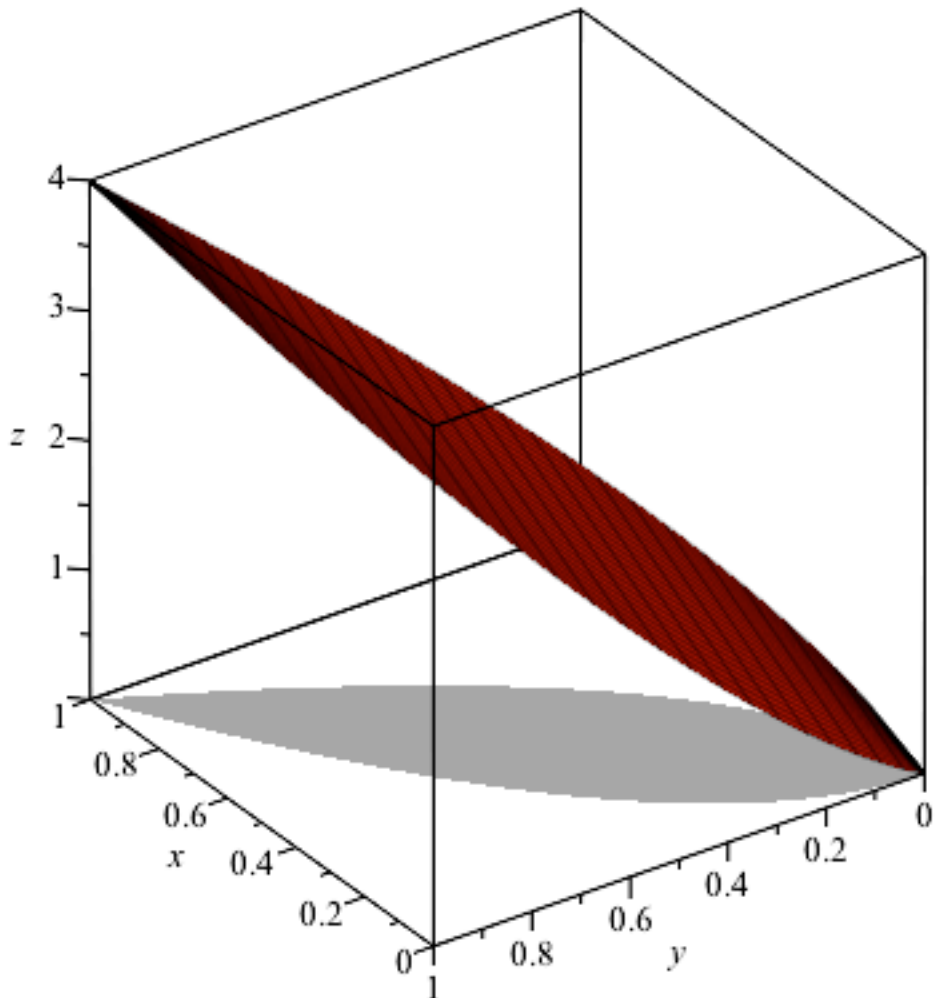
$$= \int_0^1 \left(3\,x\Big|_{x=y^2\ ..\sqrt{y}}\right) dy$$

$$= \int_0^1 \left(3\sqrt{y}-3\,y^2\right) dy$$

$$= \left(2\,y^{3\ /\ 2}-y^3\right)\Big|_{y=0\ ..1}$$

$$1 \tag{8}$$

$$display\left(plot3d\left([x,y,0],\ x=y^2\ ..\mathrm{sqrt}(y),\ y=0\ ..1,\ color="Grey",\ style=patchnogrid\right),\ plot3d\left([x,y,2\cdot x+2\cdot y],\ x=y^2\ ..\mathrm{sqrt}(y),\ y=0\ ..1,\ color="Red"\right),\ axes=boxed,\ labels=[x,y,z],\ orientation=[145,60]\right);$$



Oppgave 6a

$$F := (x, y) \rightarrow VectorField\left(\left\langle -\frac{y}{x^2 + y^2}, \frac{x}{x^2 + y^2}, 0 \right\rangle \right) :$$

$$simplify(Curl(F(x, y)).\langle 0, 0, 1 \rangle);$$

$$0 \tag{9}$$

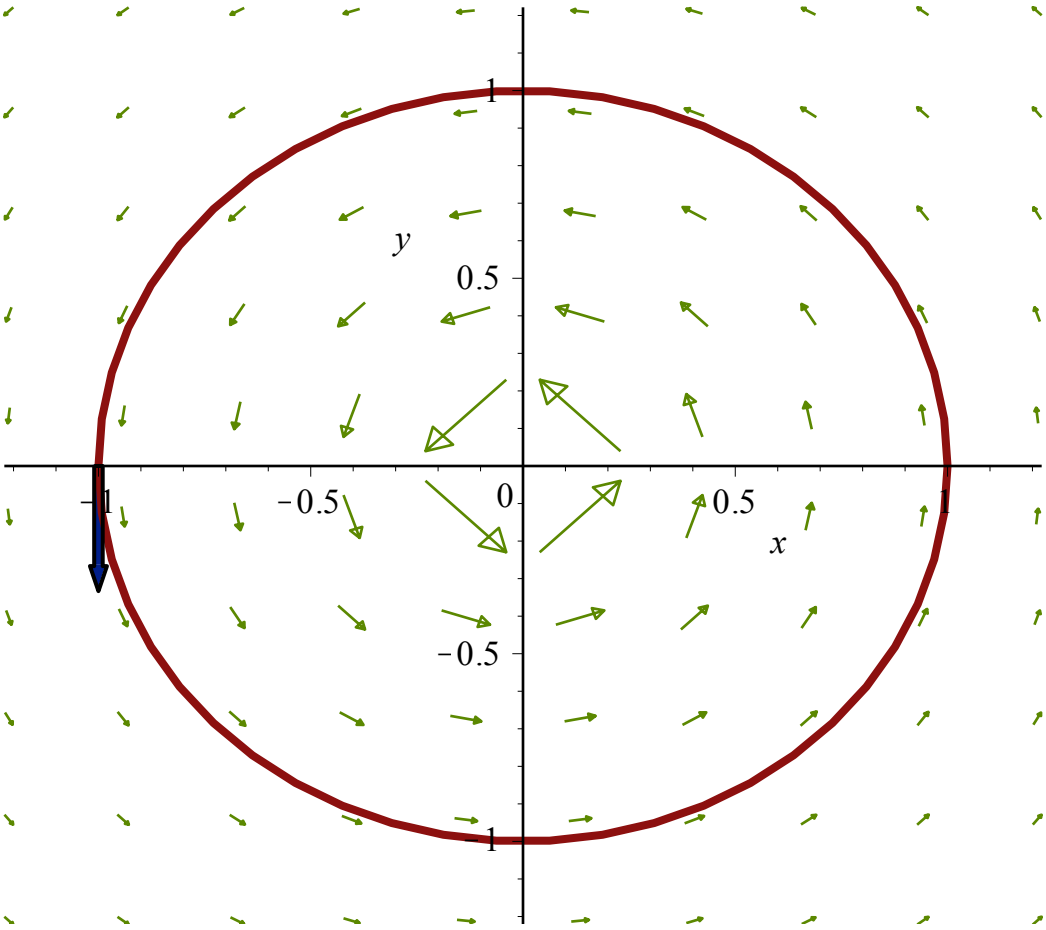
Oppgave 6b

$$F := (x, y) \rightarrow VectorField\left(\left\langle -\frac{y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right\rangle \right) :$$

$$LineInt(F(x, y), Circle(\langle 0, 0 \rangle, 1));$$

$$2\,\pi \tag{10}$$

$$LineInt(F(x, y), Circle(\langle 0, 0 \rangle, 1), fieldoptions = [arrows = slim, grid = [10, 10]], output = plot);$$



The path of integration, vector(s) tangent to the path, and vector-field arrows

Oppgave 7

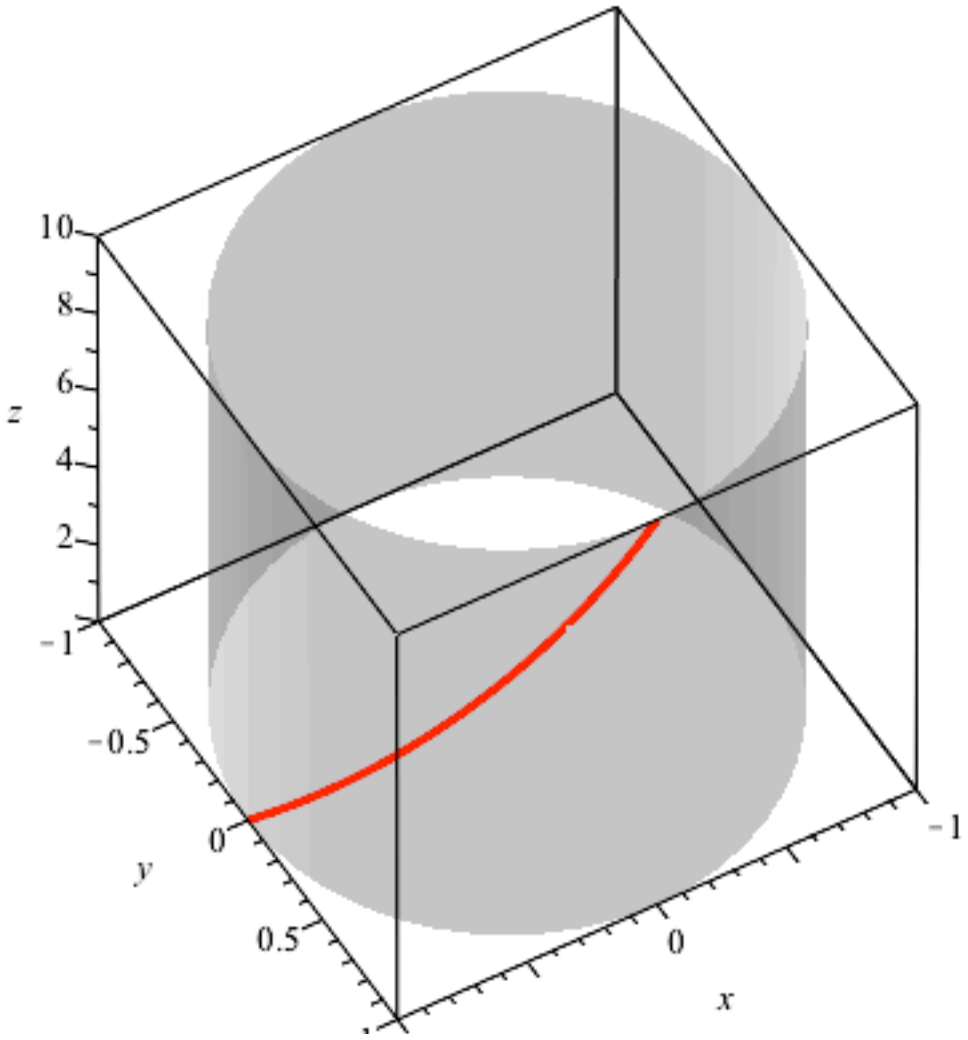
$$r := \text{theta} \to \left\langle \cos(\text{theta}),\ \sin(\text{theta}),\ \frac{2 \cdot h \cdot \text{theta}}{\text{Pi}} \right\rangle :$$

$$ArcLength\left(r(\text{theta}),\ \text{theta} = 0 .. \frac{\text{Pi}}{2} \right);$$

$$\frac{1}{2} \sqrt{\pi^2 + 4\, h^2} \tag{11}$$

$$hValue := 10 :$$

$$display\Bigg(plot3d([1, \text{theta}, z], \text{theta} = 0 .. 2 \cdot \text{Pi}, z = 0 .. hValue, coords = cylindrical, color = \text{"Grey"},$$
$$transparency = 0.8, style = patchnogrid), spacecurve\left(\left\langle \cos(\text{theta}),\ \sin(\text{theta}),\ \frac{2 \cdot hValue \cdot \text{theta}}{\text{Pi}} \right\rangle, \right.$$
$$\left. \text{theta} = 0 .. \frac{\text{Pi}}{2}, color = \text{"Red"}, thickness = 3 \right), axes = boxed, labels = [x, y, z], orientation = [60,$$
$$40] \Bigg);$$


10
8
6
z
4
2
-1
-0.5
0
y
0.5
-1
0
x