Some good commands for derivatives, differentiability, and the chain rule

If something goes bad, use

```
> restart;
```

Load the 'VectorCalculus' package – good for many things in this and other math courses:

```
> with(VectorCalculus);
```

$$[\&x, `*`, `+`, `-`, `.`, <,>, <|>, About, AddCoordinates, ArcLength, BasisFormat, Binormal, Compatibility, ConvertVector, CrossProduct, Curl, Curvature, D, Del, DirectionalDiff, Divergence, DotProduct, Flux, GetCoordinateParameters, GetCoordinates, GetNames, GetPVDescription, GetRootPoint, GetSpace, Gradient, Hessian, IsPositionVector, IsRootedVector, IsVectorField, Jacobian, Laplacian, LineInt, MapToBasis, Nabla, Norm, Normalize, PathInt, PlotPositionVector, PlotVector, PositionVector, PrincipalNormal, RadiusOfCurvature, RootedVector, ScalarPotential, SetCoordinateParameters, SetCoordinates, SpaceCurve, SurfaceInt, TNBFrame, Tangent, TangentLine, TangentPlane, TangentVector, Torsion, Vector, VectorField, VectorPotential, VectorSpace, Wronskian, diff, eval, evalVF, int, limit, series] \tag{1}$$

Define a function of, say, two variables. Although w(x,y) := x^2 + y^2 is more precise, w := x^2 + y^2 is both correct and short.

```
> w:= x^2 + y^2;
```

$$w := x^2 + y^2 \tag{2}$$

Now, to compute the gradient of w with respect to the variables (x,y):

```
> Gradient(w,[x,y]);
```

$$2\,x\bar{e}_x + 2\,y\bar{e}_y \tag{3}$$

The directional derivative in the direction v = (1,0) with respect to the variables (x,y) – which of course coincides with the first partial derivative:

```
> DirectionalDiff(w,<1,0>,[x,y]);
```

$$2\,x \tag{4}$$

Note that in Maple the vector v does not need to be normalized to unit size, whereas it has to if you do the calculation by hand. For example, the directional derivative in the direction (1,2):

```
> DirectionalDiff(w,<1,2>,[x,y]);
```

$$\frac{2}{5}\,x\sqrt{5} + \frac{4}{5}\,y\sqrt{5} \tag{5}$$

To avoid the repeating terms in the above expression, one can always try

```
> factor(%);
```

$$\frac{2}{5}\sqrt{5}\,(x + 2\,y) \tag{6}$$

To see the chain rule in action, use the 'eval' command (for 'evalute') to define a function F depending on t by F(t) = w(x(t),y(t)):

```
> F := eval(w,[x=x(t),y=y(t)]);
```

$$F := x(t)^2 + y(t)^2 \tag{7}$$

Then differentiate it with respect to t:

```
> diff(F,t);
```

$$2\,x(t)\left(\frac{\mathrm{d}}{\mathrm{d}t}\,x(t)\right) + 2\,y(t)\left(\frac{\mathrm{d}}{\mathrm{d}t}\,y(t)\right) \tag{8}$$

If the functions x(t) and y(t) are known, like if x(t) = t cos(t), y(t) = t sin(t), one can calculate this as well (the command 'simplify' can always be tried when one expects it may do some good).

```
> F := eval(w,[x=t*cos(t),y=t*sin(t)]); diff(F,t); simplify(%);
```

$$F := t^2\cos(t)^2 + t^2\sin(t)^2$$

$$2\,t\cos(t)^2 + 2\,t\sin(t)^2$$

$$2\,t \tag{9}$$

Finally, to evalute the derivative of F(t) at a point t = 3, just use 'eval' as above:

```
> diff(F,t);
```

$$2\,t\cos(t)^2 + 2\,t\sin(t)^2 \tag{10}$$

```
> eval(%,t=3);
```

$$6\cos(3)^2 + 6\sin(3)^2 \tag{11}$$

```
> simplify(%);
```

$$6 \tag{12}$$

Finally: Don't do all this for the function x^2 + y^2 . *But do use it to play around and check your calculations for more complicated expressions*. It can save you much time throughout your studies!