> $with(plots):$

> $with(Student[MultivariateCalculus]):$

> $MultiInt(1, z = 0..1-y, y = x^2..1, x = -1..1, output = steps)$

$$\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} 1\, dz\, dy\, dx$$

$$= \int_{-1}^{1}\int_{x^2}^{1} \left(z \Big|_{z = 0..1-y} \right) dy\, dx$$

$$= \int_{-1}^{1}\int_{x^2}^{1} (1-y)\, dy\, dx$$

$$= \int_{-1}^{1} \left(\left(y - \frac{1}{2} y^2 \right) \Big|_{y = x^2..1} \right) dx$$

$$= \int_{-1}^{1} \left(\frac{1}{2} - x^2 + \frac{1}{2} x^4 \right) dx$$

$$= \left(\frac{1}{2} x - \frac{1}{3} x^3 + \frac{1}{10} x^5 \right) \Big|_{x = -1..1}$$

$$\frac{8}{15} \tag{1}$$

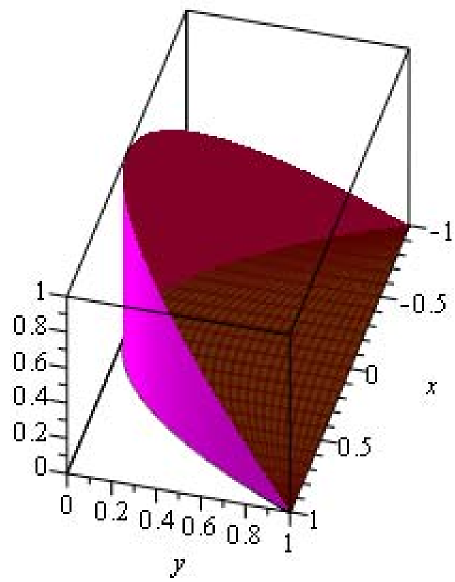
> $T1oppe := plot3d([x, y, 1-y], x = -1..1, y = x^2..1, color = \text{"Red"}, style = patchnogrid, transparency = 0.3):$

> $T1nede := plot3d([x, y, 0], x = -1..1, y = x^2..1, color = \text{"Green"}):$

> $T1side := plot3d([x, x^2, 1-y], x = -1..1, y = x^2..1, color = \text{"Magenta"}, style = patchnogrid):$

> $display(T1oppe, T1nede, T1side, scaling = constrained, axes = boxed, orientation = [15, 50], title = \text{"Integrasjonområdet til (1)"})$

Integrasjonområdet til (1)



> $MultiInt\left(1, z = 3\cdot x^2 + 3\cdot y^2 - 16 .. 9 - x^2 - y^2, y = -\mathrm{sqrt}\left(\frac{25}{4} - x^2\right) .. \mathrm{sqrt}\left(\frac{25}{4} - x^2\right), x = -\frac{5}{2} .. \frac{5}{2}, output = steps\right)$

$$\int_{-\frac{5}{2}}^{\frac{5}{2}} \int_{-\frac{\sqrt{25-4x^2}}{2}}^{\frac{\sqrt{25-4x^2}}{2}} \int_{3x^2+3y^2-16}^{9-x^2-y^2} 1 \, dz \, dy \, dx$$

$$= \int_{-\frac{5}{2}}^{\frac{5}{2}} \int_{-\frac{\sqrt{25-4x^2}}{2}}^{\frac{\sqrt{25-4x^2}}{2}} \left(z \Big|_{z = 3x^2 + 3y^2 - 16 .. 9 - x^2 - y^2} \right) dy \, dx$$

$$= \int_{-\frac{5}{2}}^{\frac{5}{2}} \int_{-\frac{\sqrt{25-4x^2}}{2}}^{\frac{\sqrt{25-4x^2}}{2}} \left(25 - 4x^2 - 4y^2\right) dy \, dx$$

$$= \int_{-\frac{5}{2}}^{\frac{5}{2}} \left(\left(25y - 4x^2 y - \frac{4}{3} y^3\right) \Bigg|_{y = -\frac{\sqrt{25-4x^2}}{2} .. \frac{\sqrt{25-4x^2}}{2}} \right) dx$$

$$= \int_{-\frac{5}{2}}^{\frac{5}{2}} \left(25\sqrt{25-4x^2} - 4x^2\sqrt{25-4x^2} - \frac{\left(25-4x^2\right)^{3/2}}{3} \right) dx$$

$$= \left(\frac{25x\sqrt{25-4x^2}}{4} + \frac{625 \arcsin\left(\frac{2x}{5}\right)}{8} + \frac{x\left(25-4x^2\right)^{3/2}}{6} \right) \Bigg|_{x = -\frac{5}{2} .. \frac{5}{2}}$$

$$\frac{625}{8} \pi \qquad \textbf{(2)}$$

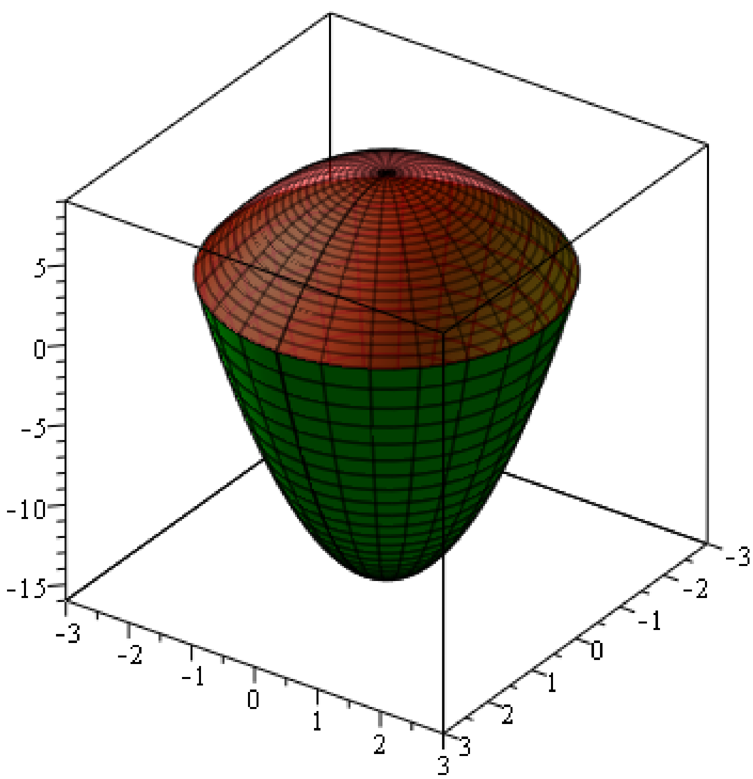
> $T2ovre := plot3d\left(\left[r, \text{theta}, 9 - r^2\right], r = 0 .. \frac{5}{2}, \text{theta} = 0 .. 2 \cdot \text{Pi}, coords = cylindrical, color\right.$

= "Red", *transparency* = 0.4) :

> $T2nedre := plot3d\left([r, \text{theta}, 3 \cdot r^2 - 16], r = 0 .. \frac{5}{2}, \text{theta} = 0 .. 2 \cdot \text{Pi}, coords = cylindrical, color = \text{"Green"} \right)$:

> *display*(*T2ovre*, *T2nedre*, *view* = [−3 ..3, −3 ..3, −16 ..9], *axes* = *boxed*, *orientation* = [35, 60], *title* = "Integrasjonsområdet til (2)")

Integrasjonsområdet til (2)



> $MultiInt\left(\exp\left(x^3\right), y = 0 .. x, x = z .. 1, z = 0 .. 1, output = steps\right)$

$$\int_0^1\int_z^1\int_0^x e^{x^3}\,dy\,dx\,dz$$

$$= \int_0^1\int_z^1 \left(e^{x^3} y \Big|_{y=0..x} \right) dx\,dz$$

$$= \int_0^1\int_z^1 e^{x^3} x\,dx\,dz$$

$$= \int_0^1 \left(-\frac{(-1)^{1/3}\left(\frac{x^2(-1)^{2/3}\Gamma\left(\frac{2}{3}\right)}{(-x^3)^{2/3}} - \frac{x^2(-1)^{2/3}\Gamma\left(\frac{2}{3}, -x^3\right)}{(-x^3)^{2/3}}\right)}{3} \Bigg|_{x=z..1} \right) dz$$

$$= \int_0^1\int_z^1 e^{x^3} x\,dx\,dz$$

$$= \left(z \left(\int_z^1 e^{x^3} x\,dx \right) + \frac{e^{z^3}}{3} \right) \Bigg|_{z=0..1}$$

$$\int_0^1\int_z^1 e^{x^3} x\,dx\,dz \qquad \textbf{(3)}$$

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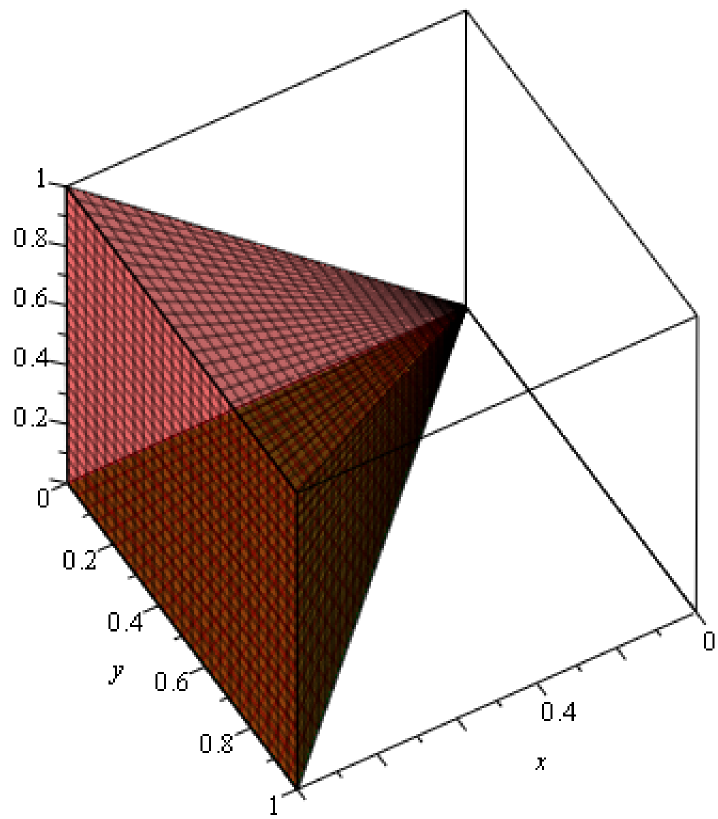
> evalf(%)
```

$$0.5727606094 \qquad \textbf{(4)}$$

```
> T3oppe := plot3d([x, y, x], x = 0..1, y = 0..x, color = "Red", transparency = 0.4) :
> T3nede := plot3d([x, y, 0], x = 0..1, y = 0..x, color = "Green") :
> T3fremme := plot3d([1, y, z], y = 0..1, z = 0..1, color = "Red", transparency = 0.4) :
> T3venstreside := plot3d([x, 0, z], x = 0..1, z = 0..x, color = "Red", transparency = 0.4) :
> T3hoyreside := plot3d([x, x, z], x = 0..1, z = 0..x, color = "Red", transparency = 0.4) :
> display(T3oppe, T3nede, T3fremme, T3venstreside, T3hoyreside, scaling = constrained, axes
      = boxed, orientation = [60, 40], title = "Integrasjonsområdet til (3) og (5)")
```

Integrasjonsområdet til (3) og (5)



> $MultiInt\left(\exp\left(x^3\right), z = 0..x, y = 0..x, x = 0..1, output = steps\right)$

$$\int_0^1\int_0^x\int_0^x e^{x^3}\,dz\,dy\,dx$$

$$= \int_0^1\int_0^x \left(e^{x^3} z\Big|_{z=0..x}\right) dy\,dx$$

$$= \int_0^1\int_0^x e^{x^3} x\,dy\,dx$$

$$= \int_0^1 \left(e^{x^3} x\,y\Big|_{y=0..x}\right) dx$$

$$= \int_0^1 e^{x^3} x^2\,dx$$

$$= \frac{e^{x^3}}{3}\Big|_{x=0..1}$$

$$-\frac{1}{3}+\frac{1}{3}e \qquad (5)$$

> *evalf*(%)

$$0.5727606093 \qquad (6)$$

>