Start

```
restart;
with(plots);
```

[animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot] **(1.1)**

```
with(Student[VectorCalculus]);
```

[&x, `*`, `+`, `-`, `.`, <,>, <|>, About, ArcLength, BasisFormat, Binormal, ConvertVector, CrossProduct, Curl, Curvature, D, Del, DirectionalDiff, Divergence, DotProduct, FlowLine, Flux, GetCoordinates, GetPVDescription, GetRootPoint, GetSpace, Gradient, Hessian, IsPositionVector, IsRootedVector, IsVectorField, Jacobian, Laplacian, LineInt, MapToBasis, Nabla, Norm, Normalize, PathInt, PlotPositionVector, PlotVector, PositionVector, PrincipalNormal, RadiusOfCurvature, RootedVector, ScalarPotential, SetCoordinates, SpaceCurve, SpaceCurveTutor, SurfaceInt, TNBFrame, Tangent, TangentLine, TangentPlane, TangentVector, Torsion, Vector, VectorField, VectorFieldTutor, VectorPotential, VectorSpace, diff, evalVF, int, limit, series] **(1.2)**

```
with(Student[MultivariateCalculus]);
```

[ApproximateInt, ApproximateIntTutor, CenterOfMass, ChangeOfVariables, CrossSection, CrossSectionTutor, Del, DirectionalDerivative, DirectionalDerivativeTutor, FunctionAverage, Gradient, GradientTutor, Jacobian, LagrangeMultipliers, MultiInt, Nabla, Revert, SecondDerivativeTest, SurfaceArea, TaylorApproximation, TaylorApproximationTutor] **(1.3)**

```
with(Student[LinearAlgebra]);
```

[&x, `.`, AddRow, AddRows, Adjoint, ApplyLinearTransformPlot, BackwardSubstitute, BandMatrix, Basis, BilinearForm, CharacteristicMatrix, CharacteristicPolynomial, ColumnDimension, ColumnSpace, CompanionMatrix, ConstantMatrix, ConstantVector, CrossProductPlot, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, EigenPlot, EigenPlotTutor, Eigenvalues, EigenvaluesTutor, Eigenvectors, EigenvectorsTutor, Equal, GaussJordanEliminationTutor, GaussianElimination, GaussianEliminationTutor, GenerateEquations, GenerateMatrix, GramSchmidt, HermitianTranspose, Id, IdentityMatrix, IntersectionBasis, InverseTutor, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, LUDecomposition, LeastSquares, LeastSquaresPlot, LinearSolve, LinearSolveTutor, LinearSystemPlot, LinearSystemPlotTutor, LinearTransformPlot, LinearTransformPlotTutor, MatrixBuilder, MinimalPolynomial, Minor, MultiplyRow, Norm, Normalize, NullSpace, Pivot, PlanePlot, ProjectionPlot, QRDecomposition, RandomMatrix, RandomVector, **(1.4)**

Rank, *ReducedRowEchelonForm*, *ReflectionMatrix*, *RotationMatrix*, *RowDimension*, *RowSpace*, *SetDefault*, *SetDefaults*, *SumBasis*, *SwapRow*, *SwapRows*, *Trace*, *Transpose*, *UnitVector*, *VectorAngle*, *VectorSumPlot*, *ZeroMatrix*, *ZeroVector*]

```
with(Student[Calculus1]);
```

[*AntiderivativePlot*, *AntiderivativeTutor*, *ApproximateInt*, *ApproximateIntTutor*, *ArcLength*, *ArcLengthTutor*, *Asymptotes*, *Clear*, *CriticalPoints*, *CurveAnalysisTutor*, *DerivativePlot*, *DerivativeTutor*, *DiffTutor*, *ExtremePoints*, *FunctionAverage*, *FunctionAverageTutor*, *FunctionChart*, *FunctionPlot*, *GetMessage*, *GetNumProblems*, *GetProblem*, *Hint*, *InflectionPoints*, *IntTutor*, *Integrand*, *InversePlot*, *InverseTutor*, *LimitTutor*, *MeanValueTheorem*, *MeanValueTheoremTutor*, *NewtonQuotient*, *NewtonsMethod*, *NewtonsMethodTutor*, *PointInterpolation*, *RiemannSum*, *RollesTheorem*, *Roots*, *Rule*, *Show*, *ShowIncomplete*, *ShowSolution*, *ShowSteps*, *Summand*, *SurfaceOfRevolution*, *SurfaceOfRevolutionTutor*, *Tangent*, *TangentSecantTutor*, *TangentTutor*, *TaylorApproximation*, *TaylorApproximationTutor*, *Understand*, *Undo*, *VolumeOfRevolution*, *VolumeOfRevolutionTutor*, *WhatProblem*] **(1.5)**

Eksempel 1

Vis at rotasjonsfeltet <-y, x, 0> er divergensfri

```
S1 := (x, y, z) → ⟨-y, x, 0⟩;
```

$$(x, y, z) \rightarrow \mathit{Student:-VectorCalculus:-<,>}(\mathit{Student:-VectorCalculus:-`-`}(y), x, 0) \quad \textbf{(2.1)}$$

Divergens

```
Divergence(S1);
```

$$(x, y, z) \rightarrow 0 \quad \textbf{(2.2)}$$

Eksempel 3

Regn ut divergensen til radialfeltet <x,y,z>

```
R1 := (x, y, z) → ⟨x, y, z⟩;
```

$$(x, y, z) \rightarrow \mathit{Student:-VectorCalculus:-<,>}(x, y, z) \quad \textbf{(3.1)}$$

Divergens

```
Divergence(R1);
```

$$(x, y, z) \rightarrow 3 \quad \textbf{(3.2)}$$

-> Kilde i hvert punkt

Eksempel 4

a) Vis at elektrisk feltet <x,y,z> / r^3 er divergensfri untatt i origo

$$F := (x, y, z) \to \frac{\langle x, y, z \rangle}{\sqrt{x^2 + y^2 + z^2}^3};$$

$$(x, y, z) \to Student:\text{-}VectorCalculus:\text{-}<,>(x, y, z)\ \frac{1}{\left(\sqrt{x^2 + y^2 + z^2}\right)^3} \tag{4.1.1}$$

Divergens

$Divergence(F);$

$$(x, y, z) \to -\frac{3x^2}{(x^2 + y^2 + z^2)^{5/2}} + \frac{3}{(x^2 + y^2 + z^2)^{3/2}} - \frac{3y^2}{(x^2 + y^2 + z^2)^{5/2}} - \frac{3z^2}{(x^2 + y^2 + z^2)^{5/2}} \tag{4.1.2}$$

$simplify(Divergence(F)(x, y, z));$

$$0 \tag{4.1.3}$$

En litt mer om partiell deriverte

$$diff\left(\frac{1}{\sqrt{x^2 + y^2 + z^2}}, x\right);$$

$$-\frac{x}{(x^2 + y^2 + z^2)^{3/2}} \tag{4.1.4}$$

$$diff\left(\frac{x}{\sqrt{x^2 + y^2 + z^2}^3}, x\right);$$

$$-\frac{3x^2}{(x^2 + y^2 + z^2)^{5/2}} + \frac{1}{(x^2 + y^2 + z^2)^{3/2}} \tag{4.1.5}$$

b) Regn ut fluksen gjennom en vilkårlig sfære med senteret i origo

Sfære med radius a
normalvektoren = r / |r|

Sfære med a = 2

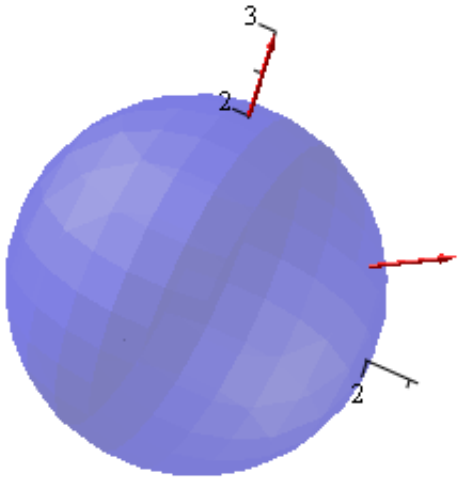
$B := implicitplot3d(subs(a = 2, x^2 + y^2 + z^2 = a^2), x = -2..2, y = -2..2, z = -2..2, style = surface, transparency = 0.7, color = blue);$

$$n := (x, y, z) \to \frac{\langle x, y, z \rangle}{\sqrt{x^2 + y^2 + z^2}};$$

$display(B, arrow(\langle 0, 0, 2 \rangle, n(0, 0, 2), color = red), arrow(\langle 0, \sqrt{3}, 1 \rangle, n(0, \sqrt{3}, 1), color = red), axes = normal, scaling = constrained);$

$$PLOT3D(\ldots)$$

$$(x, y, z) \to Student:\text{-}VectorCalculus:\text{-}<,>(x, y, z)\ \frac{1}{\sqrt{x^2 + y^2 + z^2}}$$



Fluks

F * n

$DotProduct(F(x,y,z),n(x,y,z))$;

$$\frac{x^2}{(x^2+y^2+z^2)^2}+\frac{y^2}{(x^2+y^2+z^2)^2}+\frac{z^2}{(x^2+y^2+z^2)^2} \qquad \textbf{(4.2.1)}$$

$simplify(DotProduct(F(x,y,z),n(x,y,z)))$;

$$\frac{1}{x^2+y^2+z^2} \qquad \textbf{(4.2.2)}$$

Vi ser at F * n = 1 / a^2

Fluks $= \iint_B F\cdot n\, d\sigma = \frac{1}{a^2}\iint_B d\sigma = \frac{1}{a^2}\cdot 4\cdot\pi\cdot a^2 = 4\cdot\pi$

Eksempel 5: Eksamensoppgave 2005 vår / 6

legemet T : begrenset av kuleflaten x^2+y^2+z^2 = 4 og sylinderen x^2+y^2=1

overflaten S : sylindrisk del S1, sfærisk del S2

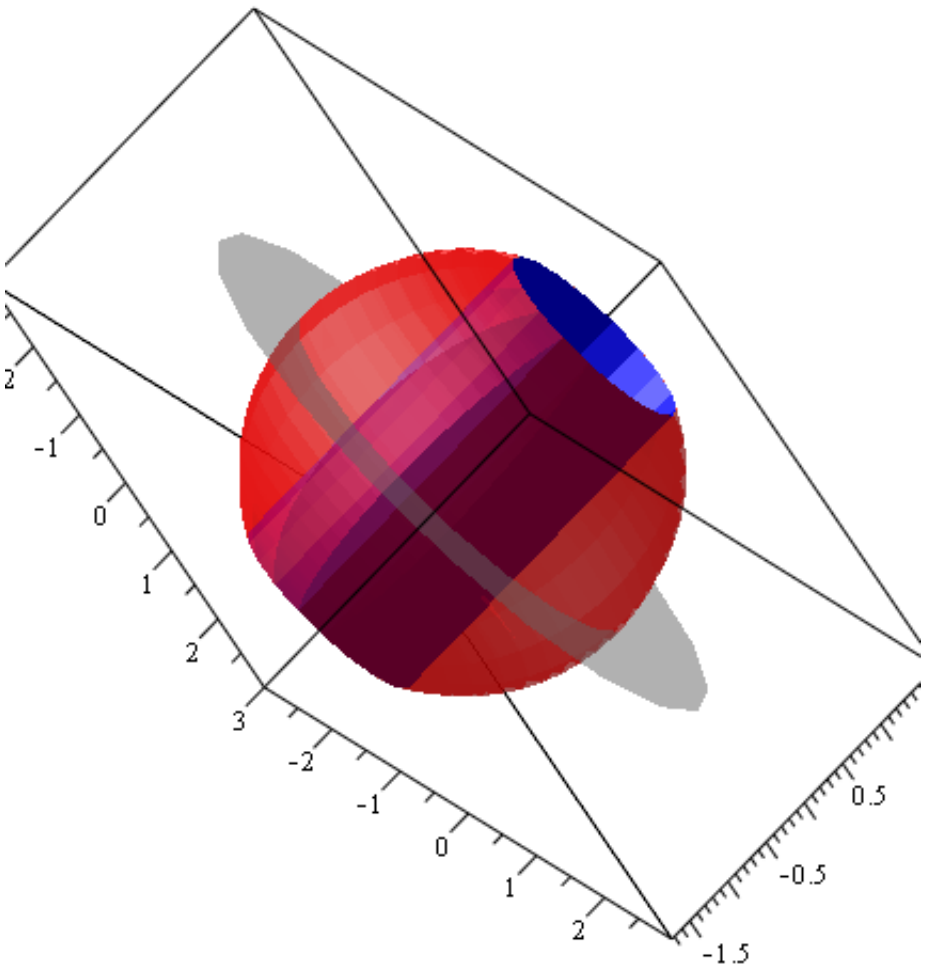
a) finn volumet til T

$$S1 := implicitplot3d\left(r^2 = 1, r = 0..2, \text{theta} = 0..2\cdot \text{Pi}, z = -\sqrt{3}..\sqrt{3}, coords = cylindrical, style = surface, color = blue, grid = [20, 20, 20]\right):$$

$$S2 := implicitplot3d\left(r^2 + z^2 = 4, r = 1..3, \text{theta} = 0..2\cdot \text{Pi}, z = -\sqrt{3}..\sqrt{3}, coords = cylindrical, style = surface, color = red, transparency = 0.3, grid = [20, 20, 20]\right):$$

$$XYplanet := implicitplot3d(z = 0, r = 0..3, \text{theta} = 0..2\cdot \text{Pi}, z = 0..1, coords = cylindrical, style = surface, color = gray, transparency = 0.5, grid = [20, 20, 20]):$$

$$display(S1, S2, XYplanet, scaling = constrained, axes = boxed);$$



Volumet

$MultiInt\left(r, z = -\sqrt{4 - r^2} \,..\sqrt{4 - r^2}, r = 1\,..2, \text{theta} = 0\,..2 \cdot \text{Pi}, output = steps\right);$

$$\int_0^{2\pi}\int_1^{2}\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_1^{2}\left(rz\Big|_{z=-\sqrt{4-r^2}\,..\sqrt{4-r^2}}\right) dr\,d\theta$$

$$= \int_0^{2\pi}\int_1^{2} 2r\sqrt{4-r^2}\,dr\,d\theta$$

$$= \int_0^{2\pi}\left(\frac{2(-2+r)(r+2)\sqrt{4-r^2}}{3}\Big|_{r=1..2}\right) d\theta$$

$$= \int_0^{2\pi} 2\sqrt{3}\,d\theta$$

$$= 2\sqrt{3}\,\theta\Big|_{\theta=0..2\pi}$$

$$4\sqrt{3}\,\pi \qquad \textbf{(5.1.1)}$$

b) finn div F, finn fluksen gjennom overflaten S når F er <x+yz, y - xz, z - e^x sin(y)>

F

$F := (x, y, z) \to \langle x + y\cdot z, y - x\cdot z, z - \exp(x)\cdot\sin(y)\rangle;$

$(x, y, z) \to Student:\text{-}VectorCalculus:\text{-}\langle,\rangle(x + y\,z, y + Student:\text{-}VectorCalculus:\text{-}\grave{}-\grave{}(z\,x), z$ **(5.2.1)**

$+ Student:\text{-}VectorCalculus:\text{-}\grave{}-\grave{}(e^x \sin(y)))$

Divergens

$Divergence(F);$

$$(x, y, z) \to 3 \qquad \textbf{(5.2.2)}$$

Divergensteoremet

$$\iint_S F\cdot n\,d\sigma = \iiint_T divF\,dx\,dy\,dz = 3\iiint_T dx\,dy\,dz = 3\cdot \text{Volumet} = 12\sqrt{3}\,\pi$$