Start

restart;
with(*plots*);

[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *densityplot*, *display*, *dualaxisplot*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *interactiveparams*, *intersectplot*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *multiple*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *rootlocus*, *semilogplot*, *setcolors*, *setoptions*, *setoptions3d*, *spacecurve*, *sparsematrixplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*] **(1.1)**

with(*Student*[*MultivariateCalculus*]);

[*ApproximateInt*, *ApproximateIntTutor*, *CenterOfMass*, *ChangeOfVariables*, *CrossSection*, *CrossSectionTutor*, *Del*, *DirectionalDerivative*, *DirectionalDerivativeTutor*, *FunctionAverage*, *Gradient*, *GradientTutor*, *Jacobian*, *LagrangeMultipliers*, *MultiInt*, *Nabla*, *Revert*, *SecondDerivativeTest*, *SurfaceArea*, *TaylorApproximation*, *TaylorApproximationTutor*] **(1.2)**

Eksempel 1

Platen R ligger i det første kvadrantet (x>=0, y>=0), mellom sirkelen x^2+y^2=1 og

$Sirkel1 := plot(\mathrm{sqrt}(1-x^2), x=0..1, color=red);$

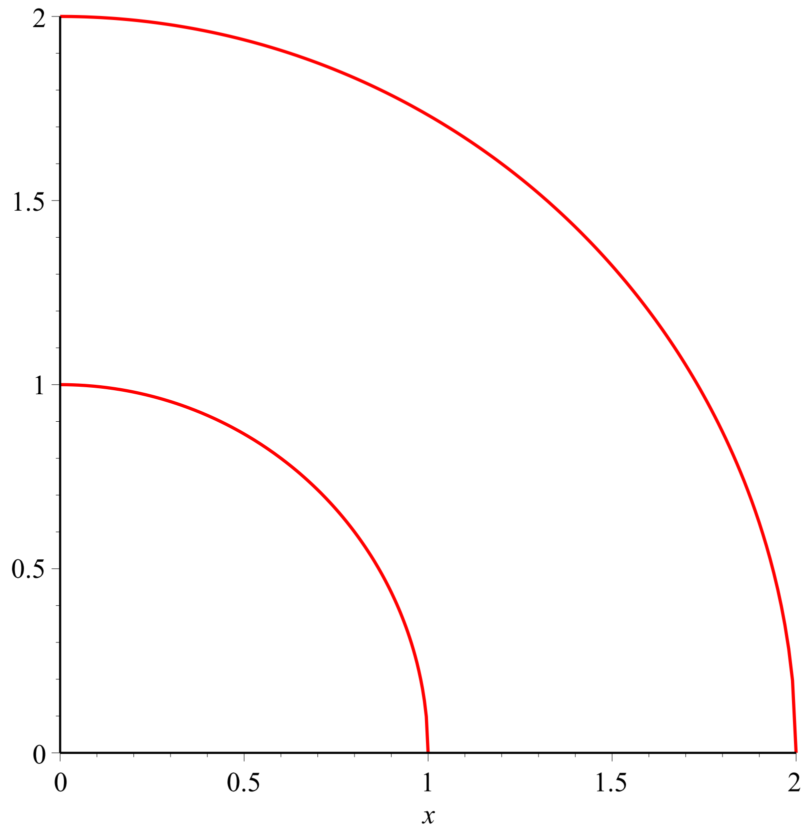
$$PLOT(...) \tag{2.1}$$

og sirkelen x^2+y^2=4

$Sirkel2 := plot(\mathrm{sqrt}(4-x^2), x=0..2, color=red);$

$$PLOT(...) \tag{2.2}$$

display(*Sirkel1*, *Sirkel2*, *axes* = *normal*, *scaling* = *constrained*);



Massetettheten er delta(x,y) = x + y i punktet (x,y) -> delta(r,Θ) = r cosΘ + r sinΘ

$\text{delta} := (r, \Theta) \rightarrow r \cdot \cos(\Theta) + r \cdot \sin(\Theta);$

$$(r, \Theta) \rightarrow r\cos(\Theta) + r\sin(\Theta) \qquad \textbf{(2.3)}$$

Massen til R er (grensene: 1 <= r <= 2, 0 <= Θ <= π/2)

$MultiInt\left(\text{delta}(r, \Theta) \cdot r,\ r = 1 ..2,\ \Theta = 0 .. \frac{\pi}{2},\ output = steps\right);$

$$\int_0^{\frac{\pi}{2}}\int_1^2 \left(r\cos(\Theta)+r\sin(\Theta)\right) r \,\mathrm{d}r\,\mathrm{d}\Theta$$

$$= \int_0^{\frac{\pi}{2}} \left(\left.\frac{\left(\cos(\Theta)+\sin(\Theta)\right) r^3}{3}\right|_{r=1..2} \right) \mathrm{d}\Theta$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{7\cos(\Theta)}{3} + \frac{7\sin(\Theta)}{3} \right) \mathrm{d}\Theta$$

$$= \left.\left(\frac{7\sin(\Theta)}{3} - \frac{7\cos(\Theta)}{3} \right)\right|_{\Theta=0..\frac{\pi}{2}}$$

$$\frac{14}{3} \tag{2.4}$$

Volumet til R er

$MultiInt\left(r, r=1..2, \Theta=0..\frac{\pi}{2}, output=steps\right);$

$$\int_0^{\frac{\pi}{2}}\int_1^2 r \,\mathrm{d}r\,\mathrm{d}\Theta$$

$$= \int_0^{\frac{\pi}{2}} \left(\left.\frac{r^2}{2}\right|_{r=1..2} \right) \mathrm{d}\Theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{3}{2}\,\mathrm{d}\Theta$$

$$= \left.\frac{3\,\Theta}{2}\right|_{\Theta=0..\frac{\pi}{2}}$$

$$\frac{3}{4}\pi \tag{2.5}$$

Den gjennomsnittlige massetettheten er

$$\frac{\frac{14}{3}}{\frac{3}{4}\pi};$$

$$\frac{56}{9\pi} \quad \textbf{(2.6)}$$

Eksempel 2

Arealet under paraboloiden z=7-x^2-y^2 og

$Paraboloide := plot3d(7 - x^2 - y^2, x = -2 ..2, y = -2 ..2);$

$$PLOT3D(...) \quad \textbf{(3.1)}$$

inni sylinderen x^2+y^2=4 og

$Sylinderen := implicitplot3d(x^2 + y^2 = 4, x = -2 ..2, y = -2 ..2, z = 0 ..6, color = gray, transparency = 0.2, style = surface);$

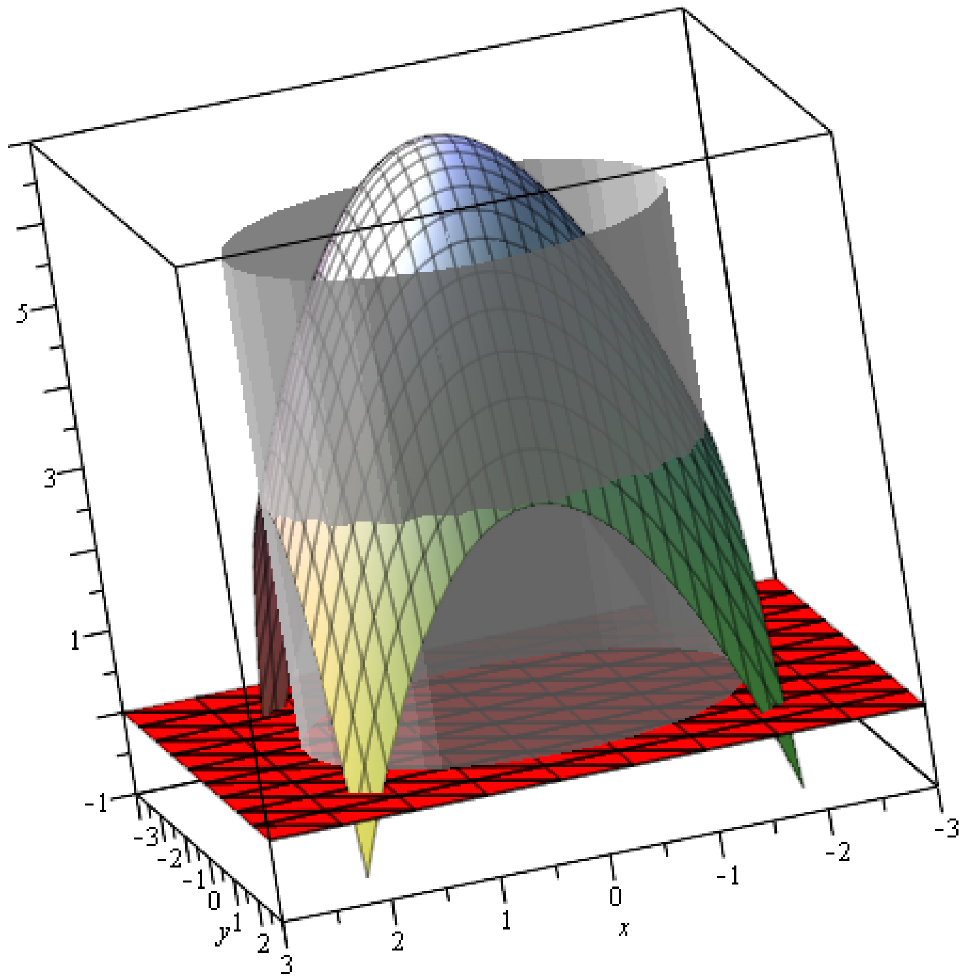
$$PLOT3D(...) \quad \textbf{(3.2)}$$

over xy-planet

$XYplanet := implicitplot3d(z = 0, x = -3 ..3, y = -3 ..3, z = 0 ..1, color = red);$

$$PLOT3D(...) \quad \textbf{(3.3)}$$

$display(Paraboloide, Sylinderen, XYplanet, axes = boxed);$



Volumet (polarkoordinater, f(r,theta), dV=r dr dΘ)

$MultiInt\left(\left(7 - r^2\right) \cdot r, r = 0..2, \text{theta} = 0..2 \cdot \text{Pi}, output = steps\right);$

$$\int_0^{2\pi}\int_0^2 (7-r^2)\, r \,\mathrm{d}r\,\mathrm{d}\theta$$

$$= \int_0^{2\pi} \left(\left(-\frac{1}{4} r^4 + \frac{7}{2} r^2 \right) \Bigg|_{r=0..2} \right) \mathrm{d}\theta$$

$$= \int_0^{2\pi} 10 \,\mathrm{d}\theta$$

$$= 10\,\theta \Big|_{\theta=0..2\pi}$$

$$20\,\pi \tag{3.4}$$