Start

Vi starter på nytt

restart **:**

Vi lader inn kommandopakken

with **(** ***Student*** **[** ***VectorCalculus*** **])**

[*&x*, `*`, `+`, `-`, `.`, *<,>*, *<|>*, *About*, *ArcLength*, *BasisFormat*, *Binormal*, *ConvertVector*, *CrossProduct*, *Curl*, *Curvature*, D, *Del*, *DirectionalDiff*, *Divergence*, *DotProduct*, *FlowLine*, *Flux*, *GetCoordinates*, *GetPVDescription*, *GetRootPoint*, *GetSpace*, *Gradient*, *Hessian*, *IsPositionVector*, *IsRootedVector*, *IsVectorField*, *Jacobian*, *Laplacian*, *LineInt*, *MapToBasis*, *Nabla*, *Norm*, *Normalize*, *PathInt*, *PlotPositionVector*, *PlotVector*, *PositionVector*, *PrincipalNormal*, *RadiusOfCurvature*, *RootedVector*, *ScalarPotential*, *SetCoordinates*, *SpaceCurve*, *SpaceCurveTutor*, *SurfaceInt*, *TNBFrame*, *Tangent*, *TangentLine*, *TangentPlane*, *TangentVector*, *Torsion*, *Vector*, *VectorField*, *VectorFieldTutor*, *VectorPotential*, *VectorSpace*, *diff*, *evalVF*, *int*, *limit*, *series*] **(1.1)**

Vi lader inn kommandopakken

with **(** ***plots*** **)**

[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *densityplot*, *display*, *dualaxisplot*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *interactiveparams*, *intersectplot*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *multiple*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *rootlocus*, *semilogplot*, *setcolors*, *setoptions*, *setoptions3d*, *spacecurve*, *sparsematrixplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*] **(1.2)**

Vi lader inn kommandopakken

with **(** ***plottools*** **)**

[*annulus*, *arc*, *arrow*, *circle*, *cone*, *cuboid*, *curve*, *cutin*, *cutout*, *cylinder*, *disk*, *dodecahedron*, *ellipse*, *ellipticArc*, *getdata*, *hemisphere*, *hexahedron*, *homothety*, *hyperbola*, *icosahedron*, *line*, *octahedron*, *parallelepiped*, *pieslice*, *point*, *polygon*, *prism*, *project*, *rectangle*, *reflect*, *rotate*, *scale*, *sector*, *semitorus*, *sphere*, *stellate*, *tetrahedron*, *torus*, *transform*, *translate*] **(1.3)**

Grafen til funksjoner av to variable

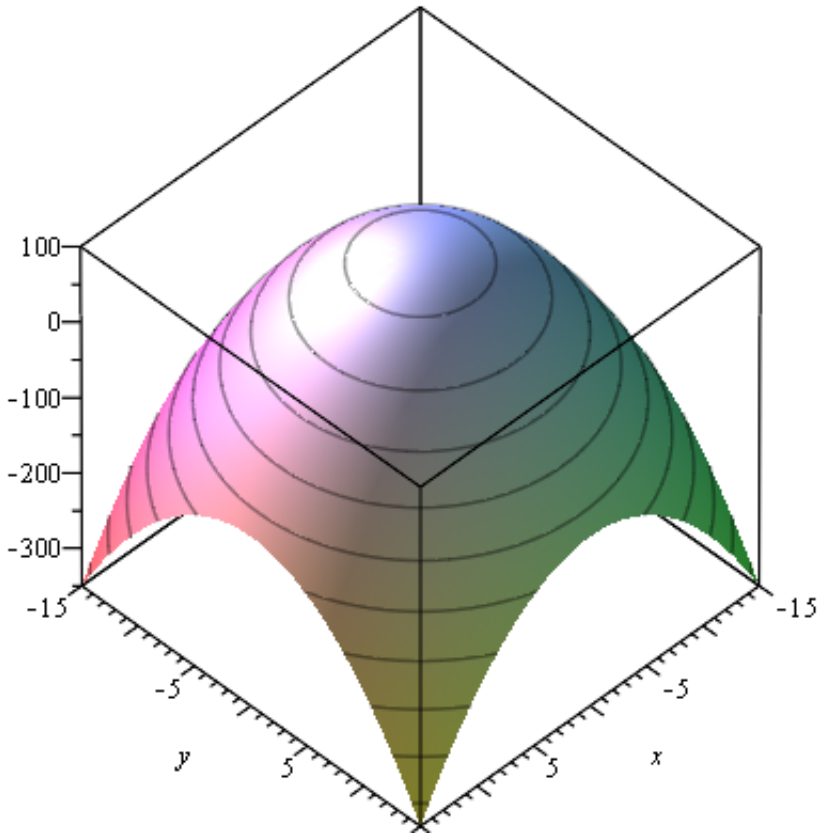
Vi skal jobbe med denne funksjonen

$f := (x, y) \to 100 - x^2 - y^2$

$(x, y) \to 100 + \mathit{Student:-VectorCalculus:-`-`}(x^2) + \mathit{Student:-VectorCalculus:-`-`}(y^2)$ **(2.1)**

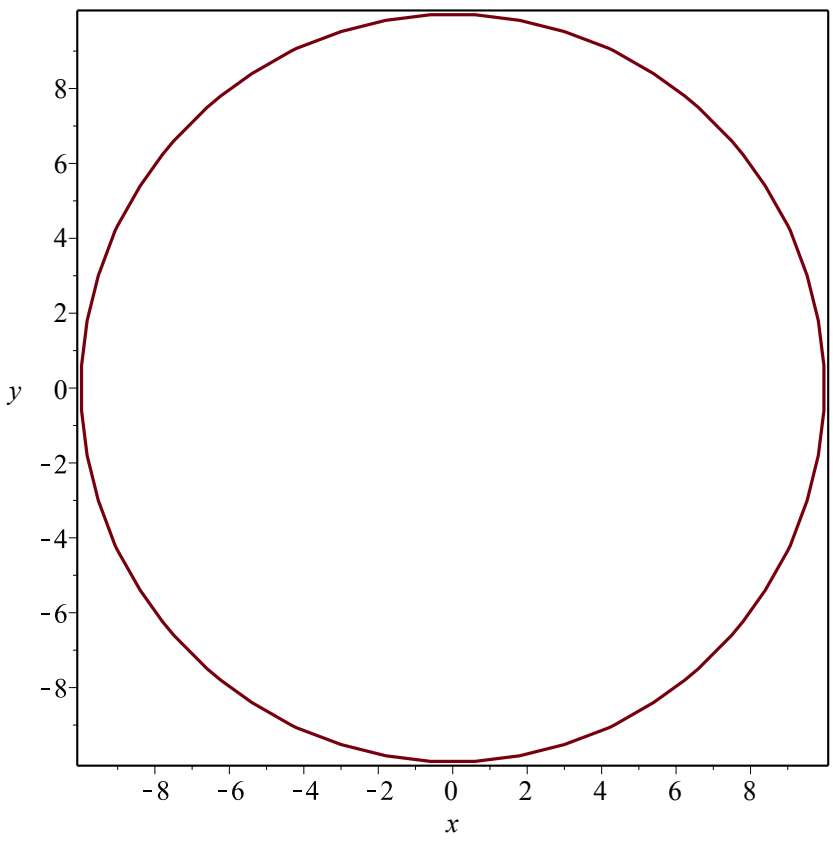
Den 3D grafen

plot3d **(** ***f*** **(** ***x*** **,** ***y*** **) ,** ***x*** **= -15 ..15,** ***y*** **= -15 ..15,** ***axes*** **=** ***boxed*** **,** ***style*** **=** ***surfacecontour*** **)**



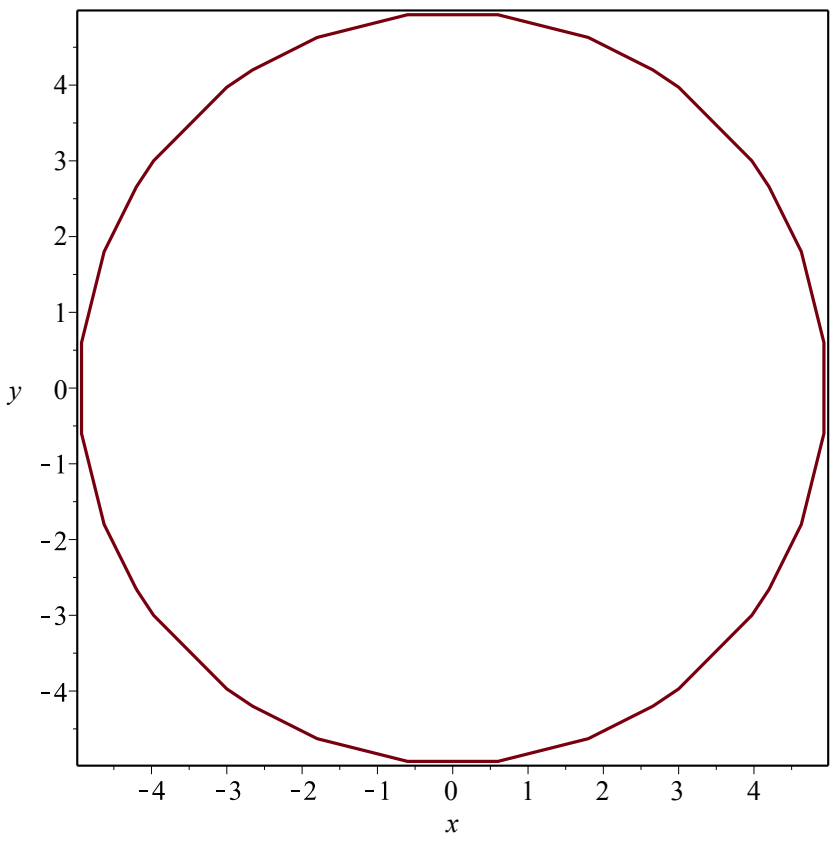
Nivåkurven f(x,y) = 0

implicitplot(*f* (*x*, *y*) = 0, *x* = - 15 ..15, *y* = - 15 ..15, *axes* = *boxed*)



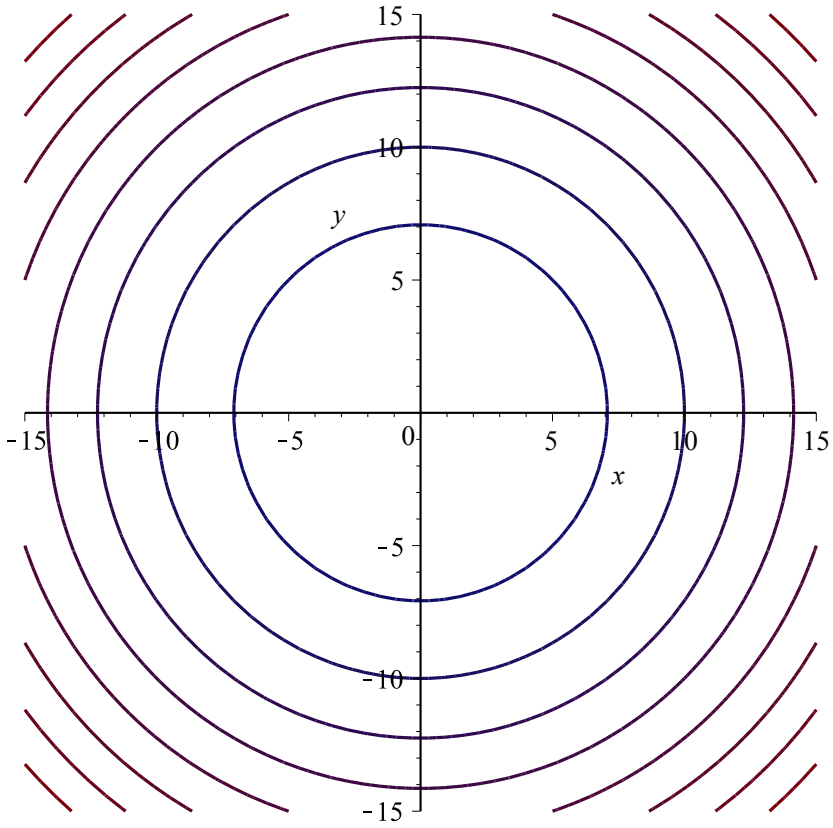
Nivåkurven f(x,y) = 75

implicitplot(*f* (*x*, *y*) = 75, *x* = - 15 ..15, *y* = - 15 ..15, *axes* = *boxed*)



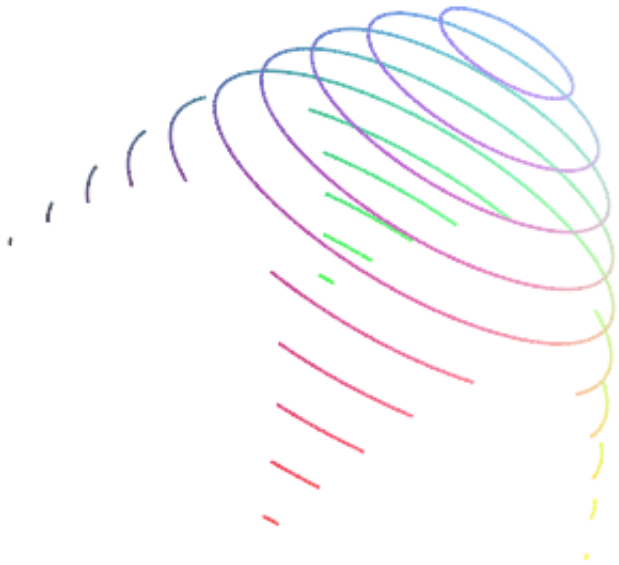
contourplot tegner noe nivåkurver

contourplot$(f(x,y), x = -15..15, y = -15..15)$



contourplot3d tegner noe konturkurver

contourplot3d$(f(x,y), x = -15..15, y = -15..15)$



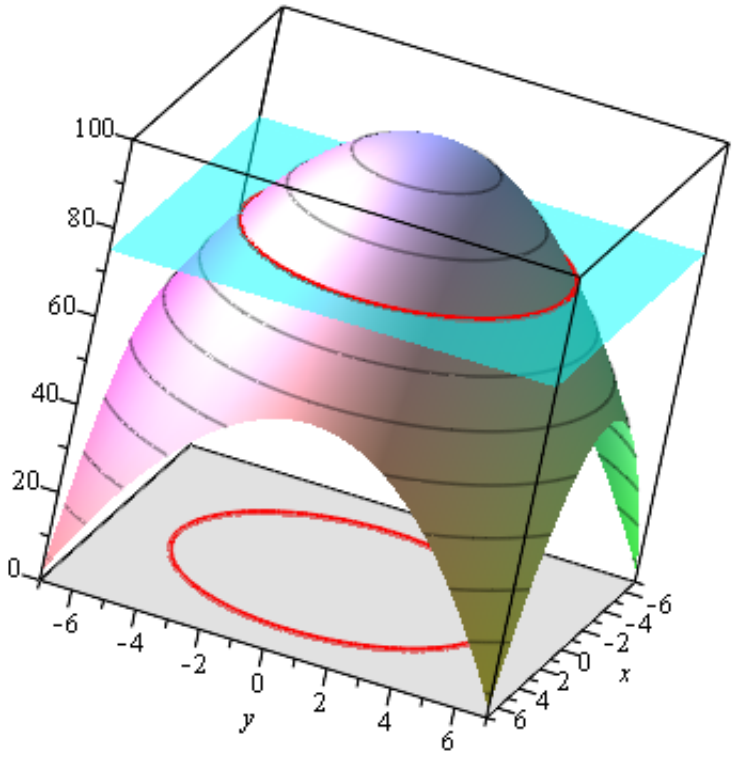
Vi lager en MAPLE-funksjon som tegner flaten for a <= x <= b, c <= y <= d og når tegner nivåkurven f(x,y) = Z

```
Flate2D := proc(f, a, b, c, d, Z)

# lokale variabler
 local Flate, zPlan, xyPlan, NivaKurve, ImbeddingNede, ImbeddingOppe, NivaKurveXYplan,
      NivaKurveZplan;

 # a <= x <= b
 # c <= y <= d
 # Vi leter etter nivåkurven som tilsvarer til f(x,y) = Z

 # flaten
Flate := plot3d(f(x, y), x = a ..b, y = c ..d, axes = boxed, style = surfacecontour);

 # zPlan
zPlan := plot3d([x, y, Z], x = a ..b, y = c ..d, style = patchnogrid, color = cyan, transparency
      = 0.5) ;

 # xyPlan
```

```
xyPlan := plot3d( [x, y, 0], x = a ..b, y = c ..d, style = patchnogrid, color = gray, transparency
      = 0.5) ;

 # Nivakurve
 NivaKurve := k→implicitplot(f(x, y) = k, x = a ..b, y = c ..d, thickness = 3, color = red, grid
      = [60, 60], axes = none);

 # Kurven på XY planet -> Nivåkurve
ImbeddingNede := transform( (x, y) → [x, y, 0]);
NivaKurveXYplan := ImbeddingNede(NivaKurve(Z) );

# Kurven på z planet -> Konturkurve
ImbeddingOppe := z→transform( (x, y) → [x, y, f(x, y) ]);
NivaKurveZplan := ImbeddingOppe(z) (NivaKurve(Z) );

 # alle sammen
display(Flate, zPlan, NivaKurveXYplan, NivaKurveZplan, xyPlan);
end;
proc( f, a, b, c, d, Z)                                                                    (2.2)
    local Flate, zPlan, xyPlan, NivaKurve, ImbeddingNede, ImbeddingOppe,
    NivaKurveXYplan, NivaKurveZplan;
    Flate := plot3d( f(x, y), x = a ..b, y = c ..d, axes = boxed, style = surfacecontour);
    zPlan := plot3d( [x, y, Z], x = a ..b, y = c ..d, style = patchnogrid, color = cyan, transparency
     = 0.5);
    xyPlan := plot3d( [x, y, 0], x = a ..b, y = c ..d, style = patchnogrid, color = gray,
    transparency = 0.5);
    NivaKurve := k→plots:-implicitplot(f(x, y) = k, x = a ..b, y = c ..d, thickness = 3, color
     = red, grid = [60, 60], axes = none);
    ImbeddingNede := plottools:-transform( (x, y) → [x, y, 0]);
    NivaKurveXYplan := ImbeddingNede(NivaKurve(Z) );
    ImbeddingOppe := z→plottools:-transform( (x, y) → [x, y, f(x, y) ]);
    NivaKurveZplan := ImbeddingOppe(z) (NivaKurve(Z) );
    plots:-display(Flate, zPlan, NivaKurveXYplan, NivaKurveZplan, xyPlan)
end proc
```

Flate2D gir dette bildet

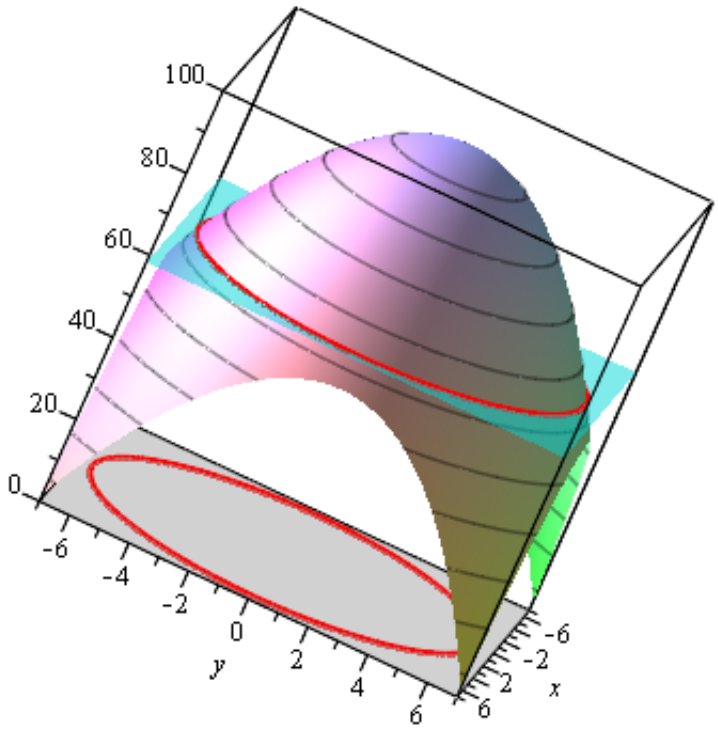
```
Flate2D(f, -7, 7, -7, 7, 75)
```



Vi lager en animasjon

$animate(Flate2D, [(f, -7, 7, -7, 7, n)], n = 0..100)$

$n = 58.333$



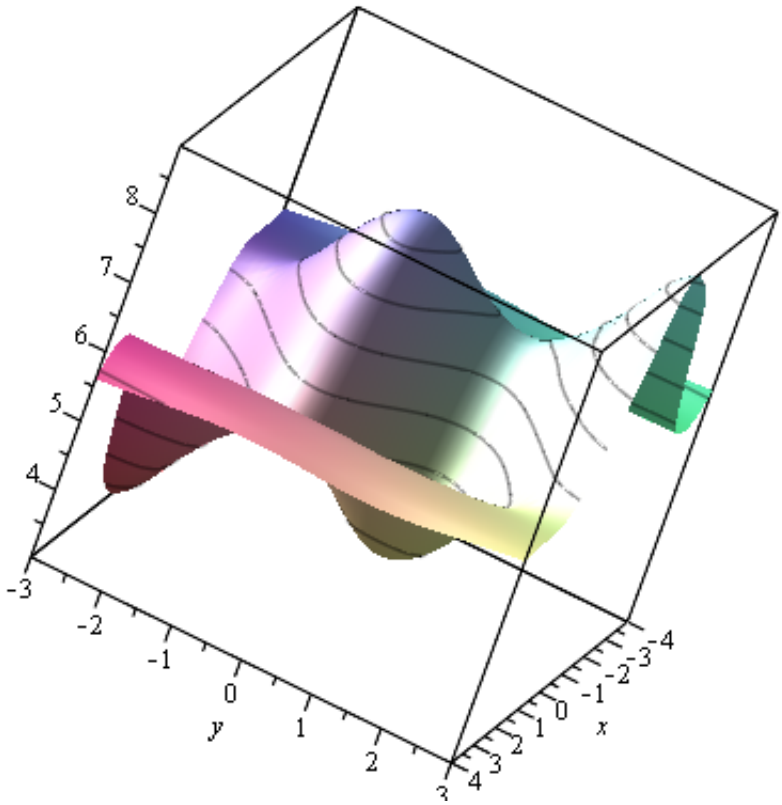
En funksjon av to variable

$$g := (x, y) \to 6 - 10 \cdot (1 + \cos(x)) \frac{y \cdot \cos(y) + x}{12 + 6 \cdot \cos(x)^2 x^2 + y^2}$$

$$(x, y) \to 6 + Student:\text{-}VectorCalculus:\text{-}`\text{-}`\left((10 + 10\cos(x))\ (y\cos(y) + x)\ \frac{1}{12 + 6\cos(x)^2 x^2 + y^2} \right) \quad \textbf{(2.3)}$$

Grafen til funksjonen

$plot3d(g(x, y), x = -4..4, y = -3..3, axes = boxed)$



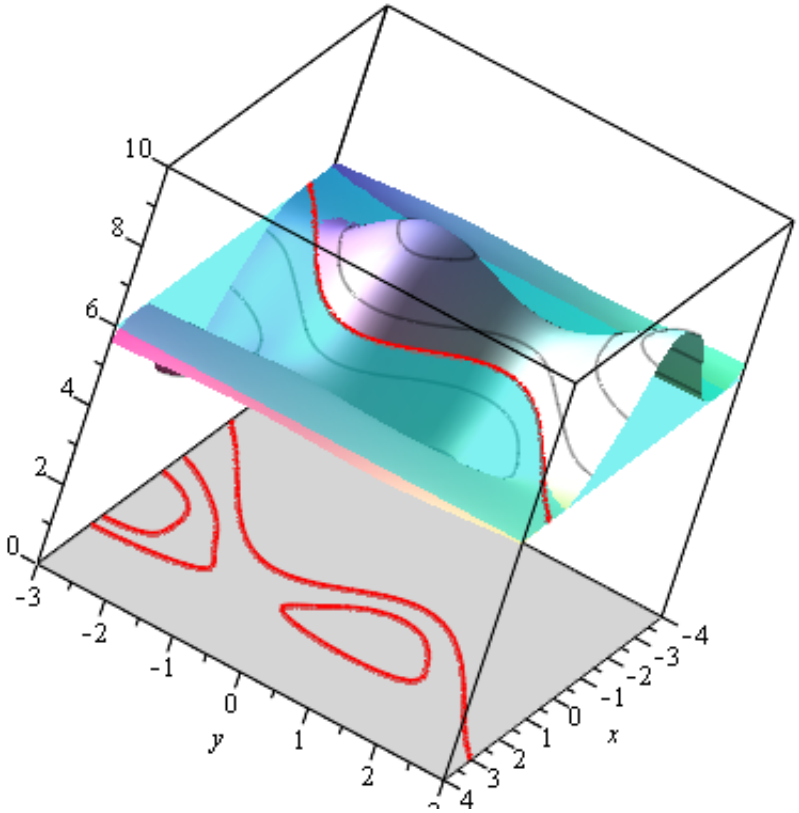
Vi lager en MAPLE-funksjon som tegner flaten samme med nivåkurvene og lager et kart

```
LagKartFraFlate := proc(f, a, b, c, d, Z1, Z2)

# lokale variabler
 local Flate, zPlan, xyPlan, NivaKurve, ImbeddingNede, ImbeddingOppe, NivaKurveXYplan,
       NivaKurveZplan;

 # a <= x <= b
 # c <= y <= d
 # Vi leter etter nivåkurvene som tilsvarer til f(x,y) = Z med Z1 <= Z <= Z2

 # flaten
Flate := plot3d(f(x, y), x = a ..b, y = c ..d, axes = boxed, style = surfacecontour);

 # zPlan
zPlan := Z→plot3d([x, y, Z], x = a ..b, y = c ..d, color = blue, style = patchnogrid, color = cyan,
       transparency = 0.5) ;

 # xyPlan
xyPlan := plot3d([x, y, 0], x = a ..b, y = c ..d, color = blue, style = patchnogrid, color = gray,
       transparency = 0.5) ;
```

```
# Nivakurve
NivaKurve := Z→implicitplot(f(x, y) = Z, x = a ..b, y = c ..d, thickness = 3, color = red, grid
    = [60, 60], axes = none);

# Kurven på XY planet -> Nivåkurve
ImbeddingNede := transform((x, y) → [x, y, 0]);

# Kurven på z planet -> Konturkurve
ImbeddingOppe := z→transform((x, y) → [x, y, f(x, y)]);

# alle sammen
display(seq(display(display(seq(ImbeddingNede(NivaKurve(t)), t = Z1 ..Z)),
    ImbeddingOppe(z)(NivaKurve(Z)), xyPlan, Flate, zPlan(Z)), Z = Z1 ..Z2), insequence
    = true, axes = boxed);
end;
proc(f, a, b, c, d, Z1, Z2)                                                    (2.4)
    local Flate, zPlan, xyPlan, NivaKurve, ImbeddingNede, ImbeddingOppe,
    NivaKurveXYplan, NivaKurveZplan;
    Flate := plot3d(f(x, y), x = a ..b, y = c ..d, axes = boxed, style = surfacecontour);
    zPlan := Z→plot3d([x, y, Z], x = a ..b, y = c ..d, color = blue, style = patchnogrid, color
    = cyan, transparency = 0.5);
    xyPlan := plot3d([x, y, 0], x = a ..b, y = c ..d, color = blue, style = patchnogrid, color
    = gray, transparency = 0.5);
    NivaKurve := Z→plots:-implicitplot(f(x, y) = Z, x = a ..b, y = c ..d, thickness = 3, color
    = red, grid = [60, 60], axes = none);
    ImbeddingNede := plottools:-transform((x, y) → [x, y, 0]);
    ImbeddingOppe := z→plottools:-transform((x, y) → [x, y, f(x, y)]);
    plots:-display(seq(plots:-display(plots:-display(seq(ImbeddingNede(NivaKurve(t)), t
    = Z1 ..Z)), ImbeddingOppe(z)(NivaKurve(Z)), xyPlan, Flate, zPlan(Z)), Z = Z1 ..Z2),
    insequence = true, axes = boxed)
end proc
```

Vi får en animasjon

```
LagKartFraFlate(g, -4, 4, -3, 3, 0, 10)
```



Grenseverdier til funksjoner av to variable

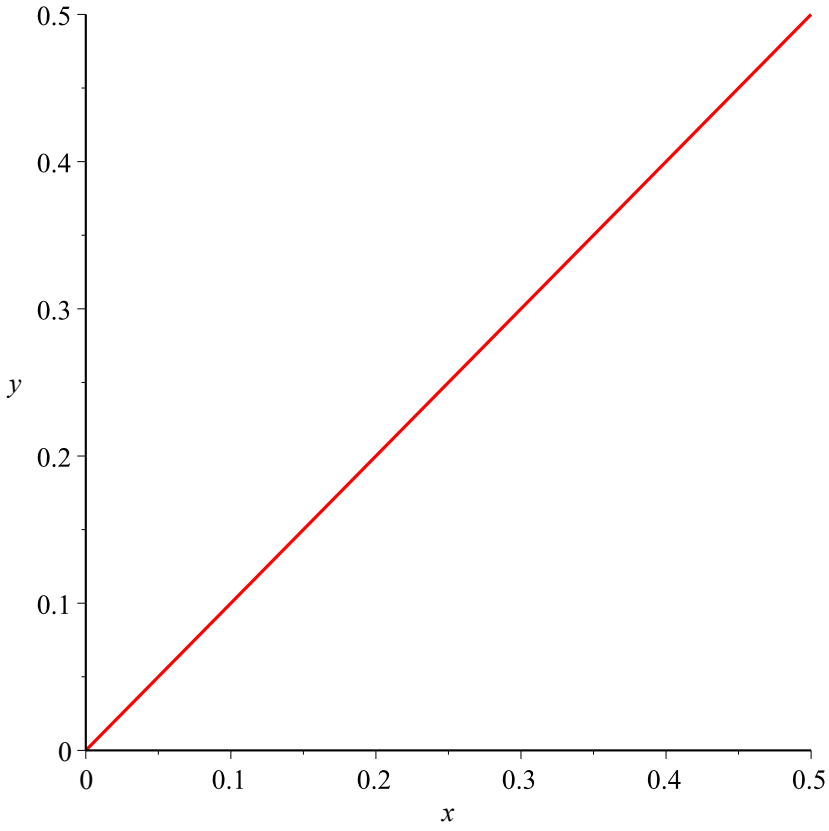
Eksempel 1

$$Eks1 := (x, y) \to \frac{x^2 - x \cdot y}{\mathrm{sqrt}(x) - \mathrm{sqrt}(y)}$$

$$(x, y) \to \left(x^2 + Student{:}\text{-}VectorCalculus{:}\text{-}\grave{}\text{-}\grave{}(x\,y)\right) \frac{1}{\sqrt{x} + Student{:}\text{-}VectorCalculus{:}\text{-}\grave{}\text{-}\grave{}\left(\sqrt{y}\right)} \quad \textbf{(3.1)}$$

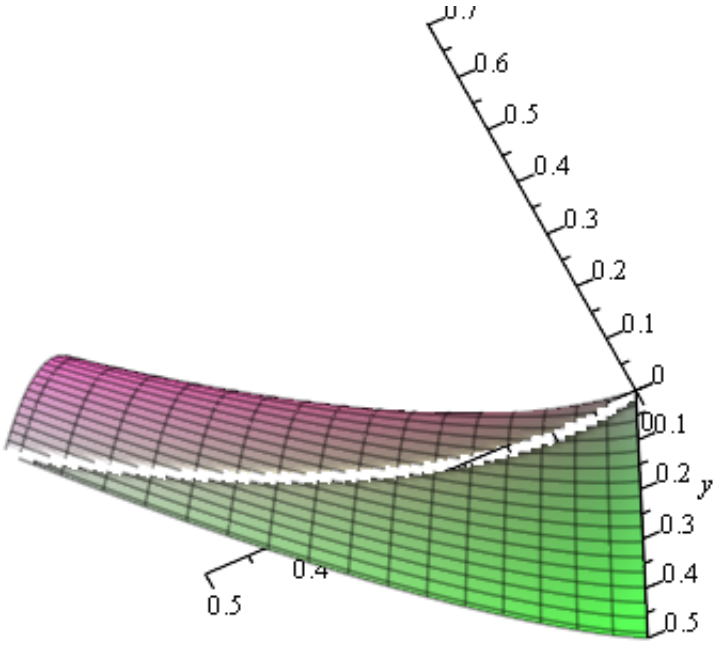
Nevneren er 0 når

$$implicitplot\left(\sqrt{x} - \sqrt{y} = 0, x = 0..0.5, y = 0..0.5, color = red\right)$$



Vi tegner grafen og ser også på bildet at det fins en grenseverdi i origo

plot3d(*Eks1*(*x*, *y*), *x* = 0 ..0.5, *y* = 0 ..0.5, *axes* = *normal*, *grid* = [50, 50])



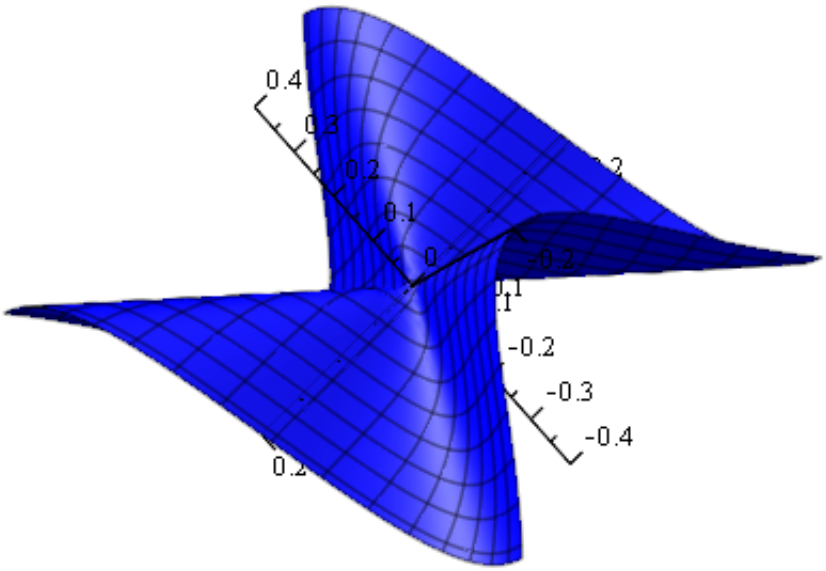
Eksempel 2

$$Eks2 := (x, y) \rightarrow \frac{4 \cdot x \cdot y^2}{x^2 + y^2}$$

$$(x, y) \rightarrow 4\, x\, y^2\, \frac{1}{x^2 + y^2} \qquad \textbf{(3.2)}$$

Grafen til funksjonen

$$plot3d(Eks2(x, y), x = -0.2 .. 0.2, y = -0.2 .. 0.2, axes = normal, grid = [50, 50], color = blue)$$


0.4
0.3
0.2
0.1
0
-0.2
-0.2
-0.3
-0.4
0.2