Vi starter på nytt
restart
Vi lader inn kommandopakken
with(*plots*)

[*animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d,*   **(1)**
    conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot,
    display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d,
    inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d,
    listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto,
    plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d,
    polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions,
    setoptions3d, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot]

Vi lader inn kommandopakken
with(*plottools*)

[*annulus, arc, arrow, circle, cone, cuboid, curve, cutin, cutout, cylinder, disk, dodecahedron,*   **(2)**
    ellipse, ellipticArc, getdata, hemisphere, hexahedron, homothety, hyperbola, icosahedron,
    line, octahedron, parallelepiped, pieslice, point, polygon, prism, project, rectangle, reflect,
    rotate, scale, sector, semitorus, sphere, stellate, tetrahedron, torus, transform, translate]

Vi skal jobbe med den følgende funksjonen
$f := (x, y) \rightarrow x^2 + 3 \cdot x \cdot y + y - 1$

$$(x, y) \rightarrow x^2 + 3\, x\, y + y - 1 \qquad \textbf{(3)}$$

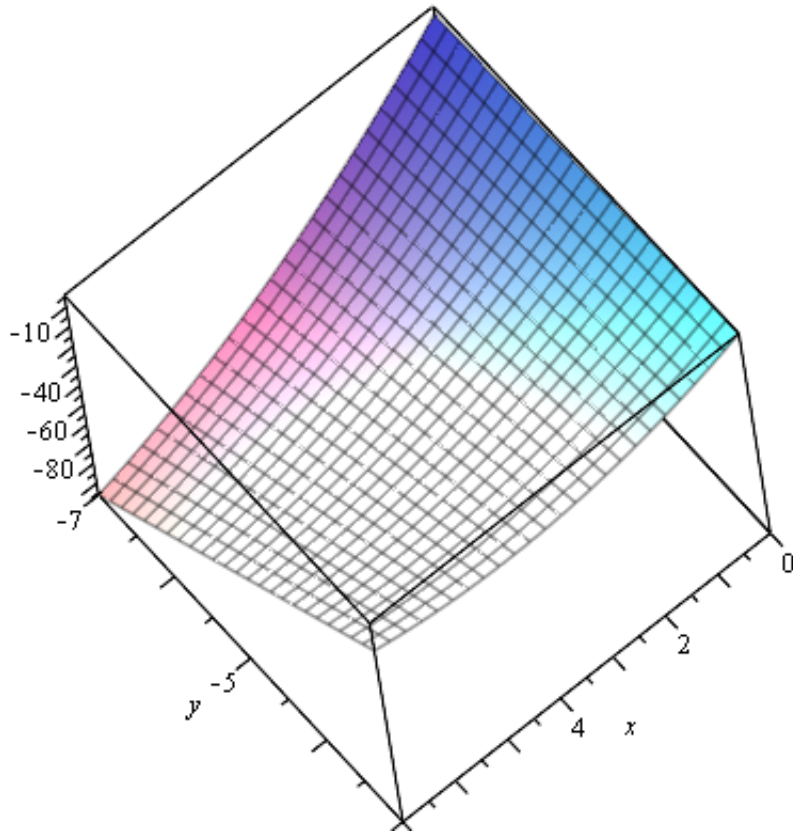Den partiellderiverte med hensyn på x
diff($f(x, y), x$)

$$2\, x + 3\, y \qquad \textbf{(4)}$$

Den partiellderiverte med hensyn på y
diff($f(x, y), y$)

$$3\, x + 1 \qquad \textbf{(5)}$$

Grafen til f
plot3d($f(x, y), x = 0\, ..7, y = -7\, ..-3, axes = boxed$)

Punktet (x0,y0)

$Punktet := sphere([4, -5, f(4, -5)], 0.05, color = blue)$

$$MESH\left(\left[\begin{array}{c} 1..49\ x\ 1..49\ x\ 1..3\ Array \\ Data\ Type:\ float_8 \\ Storage:\ rectangular \\ Order:\ C\_order \end{array}\right], COLOR(RGB, 0., 0., 1.00000000)\right)$$ (6)
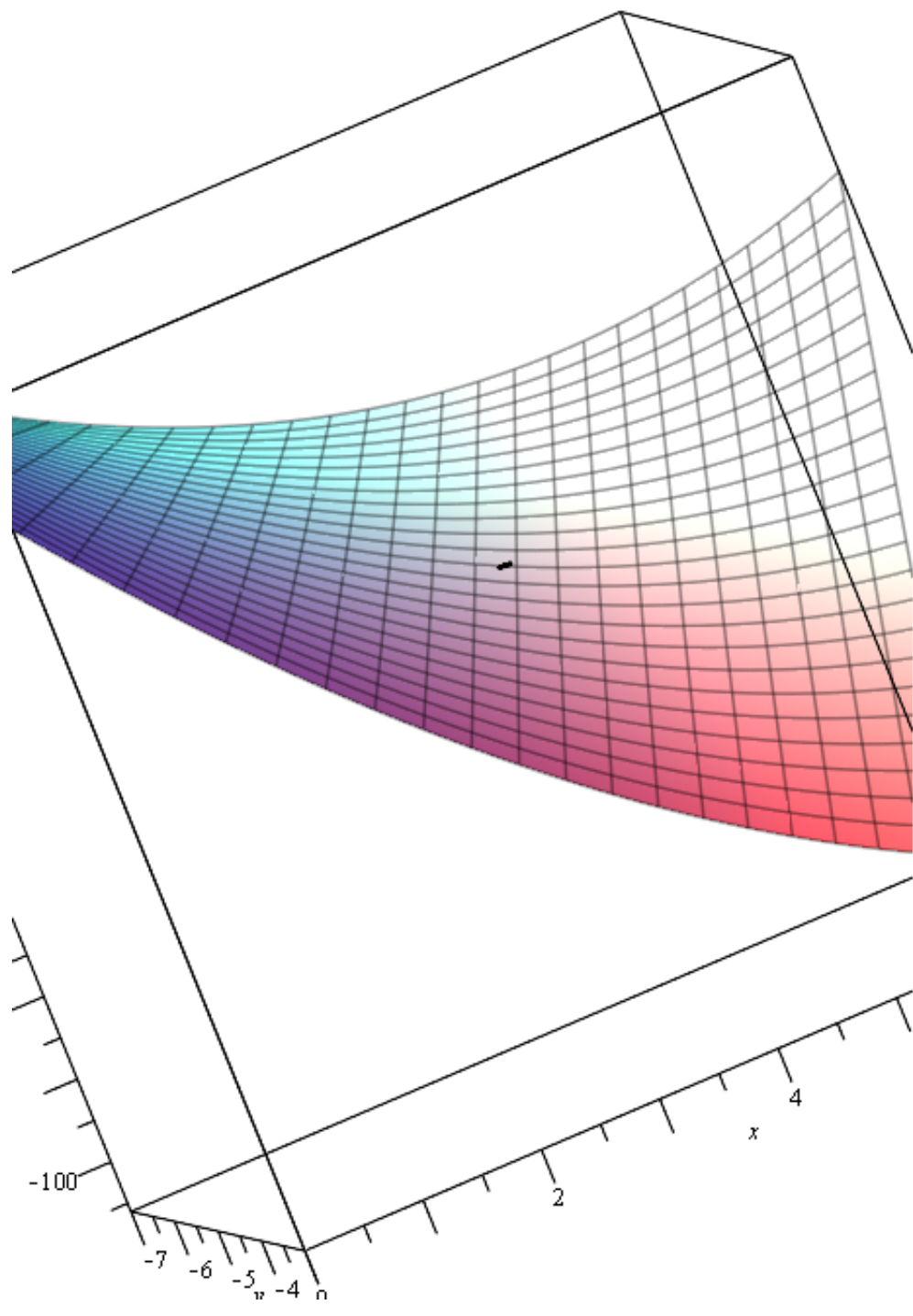
Flaten

$Flaten := plot3d(f(x, y), x = 0..7, y = -7..-3, axes = boxed)$

$$PLOT3D(...)$$ (7)

De to sammen

$display(Flaten, Punktet)$

Vi skal jobbe med den følgende funksjonen

$f := (x, y) \rightarrow \dfrac{2\,y}{y + \cos(x)}$

$$(x, y) \rightarrow \dfrac{2\,y}{y + \cos(x)} \tag{8}$$

Den partiellderiverte med hensyn på x
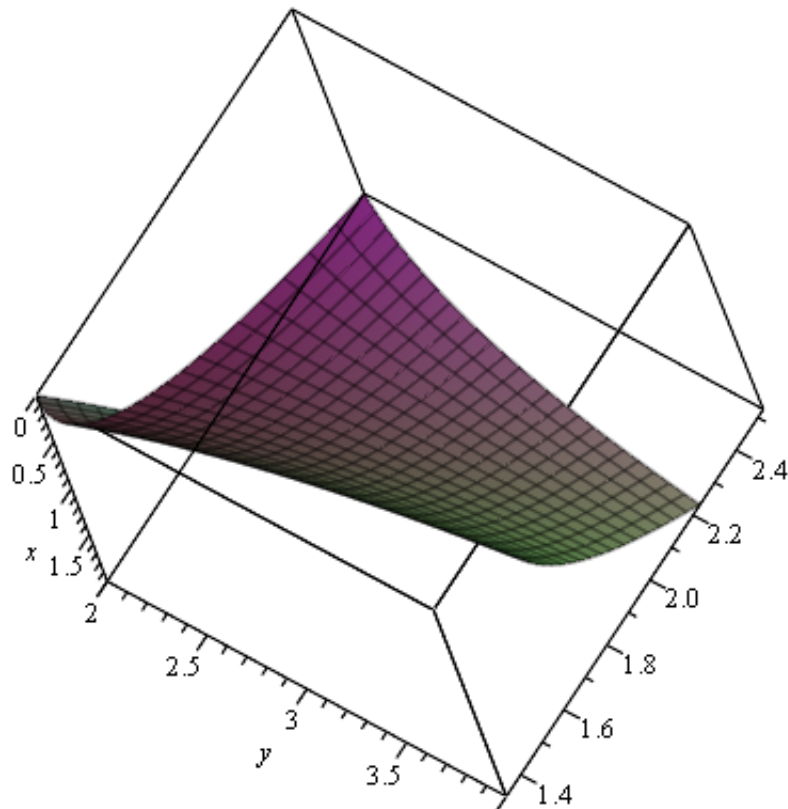$diff\,(f(x, y), x)$

$$\dfrac{2\,y\,\sin(x)}{(y + \cos(x))^2} \tag{9}$$

Den partiellderiverte med hensyn på y
$diff\,(f(x, y), y)$

$$\dfrac{2}{y + \cos(x)} - \dfrac{2\,y}{(y + \cos(x))^2} \tag{10}$$

Grafen til f
$plot3d(f(x, y), x = 0\,..2, y = 2\,..4, axes = boxed)$

Vi skal jobbe med den følgende funksjonen

$f := (x, y) \rightarrow \dfrac{x^2 + 3\,x\,y + y - 1}{x^2 + y^2 + 1}$

$$(x, y) \rightarrow \dfrac{x^2 + 3\,x\,y + y - 1}{x^2 + y^2 + 1} \tag{11}$$

Partiellderiverte
diff$(f(x, y), x)$

$$\dfrac{2\,x + 3\,y}{x^2 + y^2 + 1} - \dfrac{2\,(x^2 + 3\,x\,y + y - 1)\,x}{(x^2 + y^2 + 1)^2} \tag{12}$$

diff$(f(x, y), y)$

$$\dfrac{3\,x + 1}{x^2 + y^2 + 1} - \dfrac{2\,(x^2 + 3\,x\,y + y - 1)\,y}{(x^2 + y^2 + 1)^2} \tag{13}$$
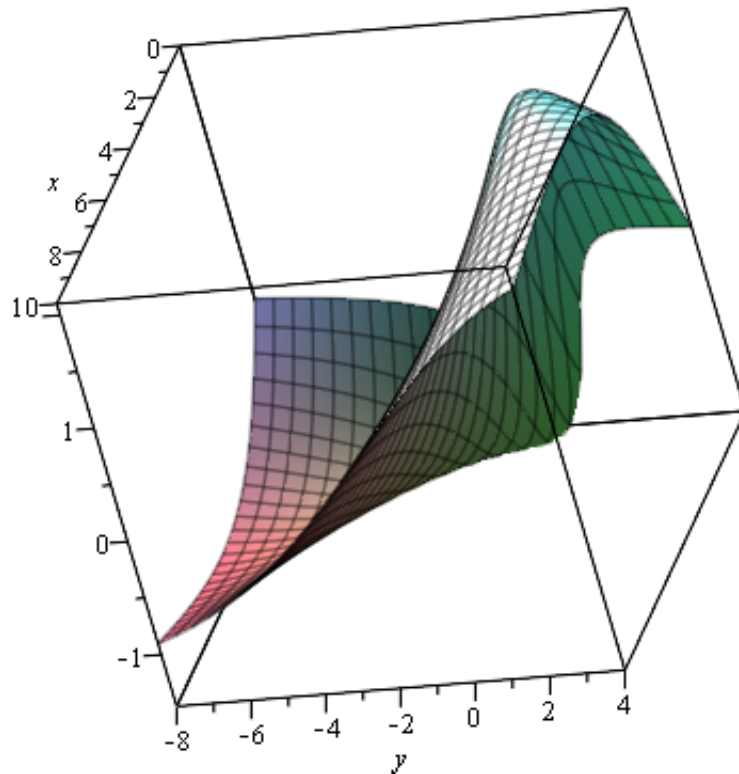
Punktet (x0,y0)
$x0 := 2$

$$2 \tag{14}$$

$y0 := -5$

$$-5 \tag{15}$$

Grafen til f
$plot3d(\,f(x, y),\, x = 0\,..10,\, y = -8\,..4,\, axes = boxed)$

Vi lager no bilder til å vise dem sammen
Flaten
Flaten := *plot3d*(*f*(*x*, *y*), *x* = 0 ..10, *y* = -8 ..4, *axes* = *boxed*)
$$PLOT3D(...)$$ **(16)**

Partiellderiverte med hensyn på x i (x0,y0)
StigningsTx := *subs*(*x* = *x0*, *y* = *y0*, diff(f(x, y), *x*))
$$-\frac{101}{450}$$ **(17)**

Partiellderiverte med hensyn på x i (x0,y0)
StigningsTy := *subs*(*x* = *x0*, *y* = *y0*, diff(f(x, y), *y*))
$$-\frac{11}{90}$$ **(18)**

Tangentlinjen paralell med x-aksen (y konstant)
TangentX := *h*→⟨*x0* + *h*, *y0*, *StigningsTx* (*x0* + *h* − *x0*) + *f*(*x0*, *y0*)⟩
$$h→⟨x0 + h, y0, StigningsTx\ (x0 + h − x0) + f(x0, y0)⟩$$

TangentLinjenX := *h*→*line*([*x0*, *y0*, *f*(*x0*, *y0*)], *convert*(*TangentX*(*h*), *list*), *color* = *red*, *thickness* = 3)
$$h→plottools\text{:-}line(\ [x0, y0, f(x0, y0)\ ], convert(\ TangentX(h), list), color = red, thickness = 3\ )$$ **(20)**

Tangentlinjen paralell med y-aksen (x konstant)

$TangentY := h \rightarrow \langle x0, y0 + h, StigningsTy\,(y0 + h - y0) + f(x0, y0) \rangle$

$$h \rightarrow \langle x0, y0 + h, StigningsTy\,(y0 + h - y0) + f(x0, y0) \rangle$$

$TangentLinjenY := h \rightarrow line(\,[x0, y0, f(x0, y0)\,], convert(TangentY(h), list), color = blue, thickness = 3\,)$

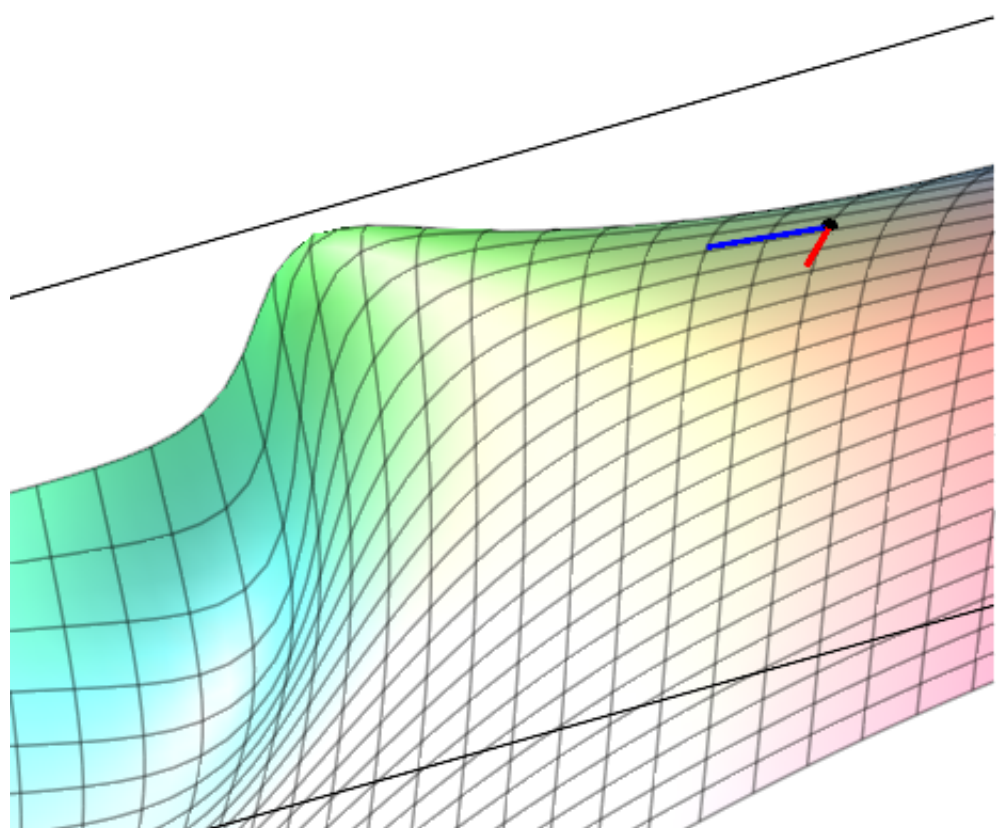$$h \rightarrow plottools\text{:-}line(\,[x0, y0, f(x0, y0)\,], convert(TangentY(h), list), color = blue, thickness = 3\,) \qquad \textbf{(22)}$$

Punktet (x0,y0)

$Punktet := sphere(\,[x0, y0, f(x0, y0)\,], 0.05, color = blue)$

$$MESH\left(\left[\begin{array}{c} \textit{1..49 x 1..49 x 1..3 Array} \\ \textit{Data Type: float}_8 \\ \textit{Storage: rectangular} \\ \textit{Order: C\_order} \end{array}\right], COLOR(RGB, 0., 0., 1.00000000)\right) \qquad \textbf{(23)}$$

Flaten, Tangentlinjene, Punktet

$display(Flaten, TangentLinjenX(1), TangentLinjenY(1), Punktet, scaling = constrained)$

Snittkurven

$Snittkurven := implicitplot3d(z = f(x, y), x = 0..10, y = -5.001..-5, z = -3..3, color = white, thickness = 5)$
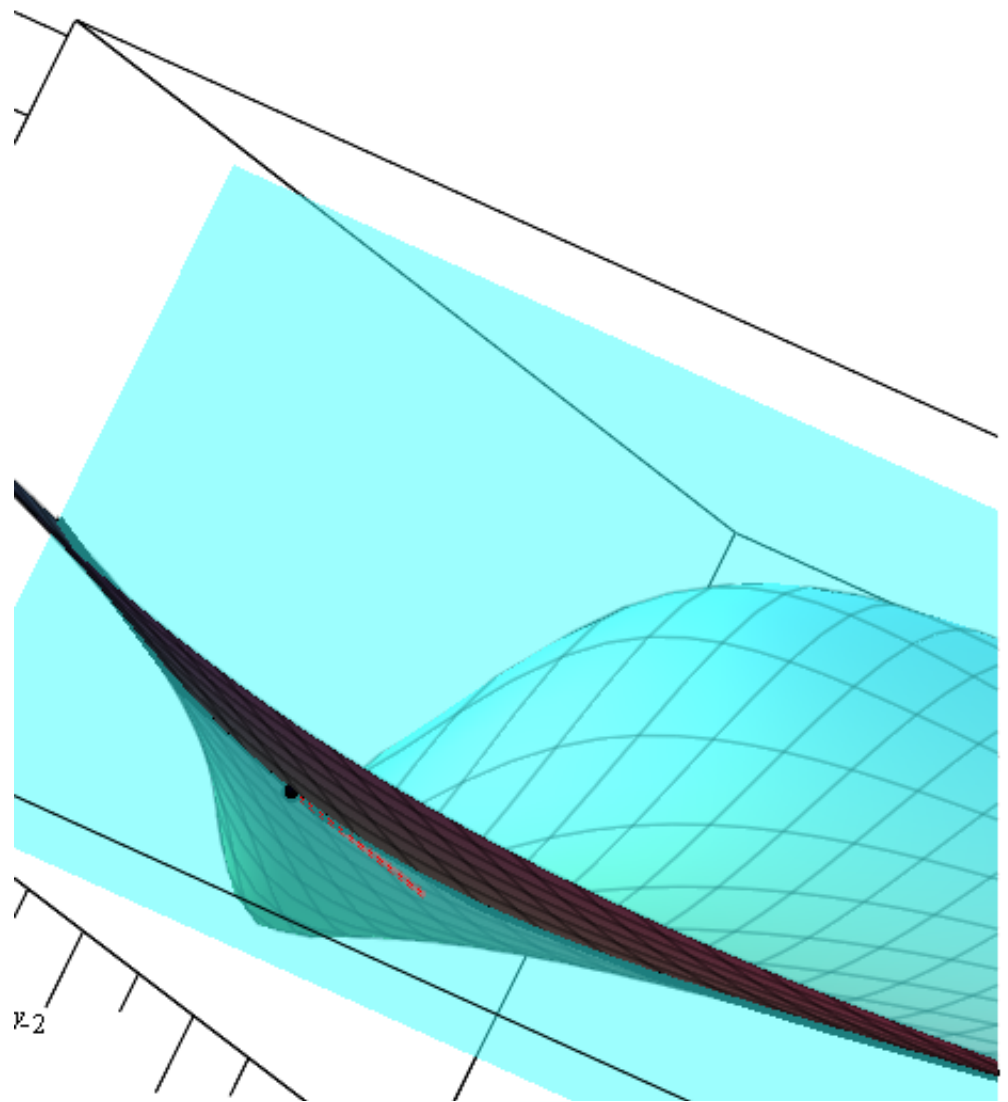
$$PLOT3D(...) \tag{24}$$

Planet y=y0

$PlanetYkonst := implicitplot3d(y = y0, x = 0..10, y = -8..4, z = -2..2, color = cyan, transparency = 0.5, style = surface)$

$$PLOT3D(...) \tag{25}$$

Flaten, Planet, Tangentlinje X, Punktet, Snittkurven

$display(Flaten, PlanetYkonst, TangentLinjenX(1), Punktet, Snittkurven, scaling = constrained)$

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Planet av Tangentlinjene

$PlanetTxTy := h \rightarrow plot3d([x0 + h1, y0 + h2, StigningsTx\, h1 + StigningsTy\, h2 + f(x0, y0)], h1 = 0 ..h,$
$\quad h2 = 0 ..h, color = cyan, transparency = 0.7)$

$h \rightarrow plot3d([x0 + h1, y0 + h2, StigningsTx\, h1 + StigningsTy\, h2 + f(x0, y0)], h1 = 0 ..h, h2 = 0$ **(26)**
$\quad ..h, color = cyan, transparency = 0.7)$

Flaten, Tangentlinjene, Planet av tangentlinjene

$display(Flaten, TangentLinjenX(1), TangentLinjenY(1), PlanetTxTy(1), Punktet, scaling$
$\quad = constrained)$