Start

```
restart;
with(plots);
```

[animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot] **(1.1)**

```
with(Student[VectorCalculus]);
```

[&x, `*`, `+`, `-`, `.`, <,>, <|>, About, ArcLength, BasisFormat, Binormal, ConvertVector, CrossProduct, Curl, Curvature, D, Del, DirectionalDiff, Divergence, DotProduct, FlowLine, Flux, GetCoordinates, GetPVDescription, GetRootPoint, GetSpace, Gradient, Hessian, IsPositionVector, IsRootedVector, IsVectorField, Jacobian, Laplacian, LineInt, MapToBasis, Nabla, Norm, Normalize, PathInt, PlotPositionVector, PlotVector, PositionVector, PrincipalNormal, RadiusOfCurvature, RootedVector, ScalarPotential, SetCoordinates, SpaceCurve, SpaceCurveTutor, SurfaceInt, TNBFrame, Tangent, TangentLine, TangentPlane, TangentVector, Torsion, Vector, VectorField, VectorFieldTutor, VectorPotential, VectorSpace, diff, evalVF, int, limit, series] **(1.2)**

```
with(Student[MultivariateCalculus]);
```

[ApproximateInt, ApproximateIntTutor, CenterOfMass, ChangeOfVariables, CrossSection, CrossSectionTutor, Del, DirectionalDerivative, DirectionalDerivativeTutor, FunctionAverage, Gradient, GradientTutor, Jacobian, LagrangeMultipliers, MultiInt, Nabla, Revert, SecondDerivativeTest, SurfaceArea, TaylorApproximation, TaylorApproximationTutor] **(1.3)**

```
with(Student[LinearAlgebra]);
```

[&x, `.`, AddRow, AddRows, Adjoint, ApplyLinearTransformPlot, BackwardSubstitute, BandMatrix, Basis, BilinearForm, CharacteristicMatrix, CharacteristicPolynomial, ColumnDimension, ColumnSpace, CompanionMatrix, ConstantMatrix, ConstantVector, CrossProductPlot, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, EigenPlot, EigenPlotTutor, Eigenvalues, EigenvaluesTutor, Eigenvectors, EigenvectorsTutor, Equal, GaussJordanEliminationTutor, GaussianElimination, GaussianEliminationTutor, GenerateEquations, GenerateMatrix, GramSchmidt, HermitianTranspose, Id, IdentityMatrix, IntersectionBasis, InverseTutor, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, LUDecomposition, LeastSquares, LeastSquaresPlot, LinearSolve, LinearSolveTutor, LinearSystemPlot, LinearSystemPlotTutor, LinearTransformPlot, LinearTransformPlotTutor, MatrixBuilder, MinimalPolynomial, Minor, MultiplyRow, Norm, Normalize, NullSpace, Pivot, PlanePlot, ProjectionPlot, QRDecomposition, RandomMatrix, RandomVector, **(1.4)**

Rank, *ReducedRowEchelonForm*, *ReflectionMatrix*, *RotationMatrix*, *RowDimension*, *RowSpace*, *SetDefault*, *SetDefaults*, *SumBasis*, *SwapRow*, *SwapRows*, *Trace*, *Transpose*, *UnitVector*, *VectorAngle*, *VectorSumPlot*, *ZeroMatrix*, *ZeroVector*]

$with(Student[Calculus1]);$

[*AntiderivativePlot*, *AntiderivativeTutor*, *ApproximateInt*, *ApproximateIntTutor*, *ArcLength*, *ArcLengthTutor*, *Asymptotes*, *Clear*, *CriticalPoints*, *CurveAnalysisTutor*, *DerivativePlot*, *DerivativeTutor*, *DiffTutor*, *ExtremePoints*, *FunctionAverage*, *FunctionAverageTutor*, *FunctionChart*, *FunctionPlot*, *GetMessage*, *GetNumProblems*, *GetProblem*, *Hint*, *InflectionPoints*, *IntTutor*, *Integrand*, *InversePlot*, *InverseTutor*, *LimitTutor*, *MeanValueTheorem*, *MeanValueTheoremTutor*, *NewtonQuotient*, *NewtonsMethod*, *NewtonsMethodTutor*, *PointInterpolation*, *RiemannSum*, *RollesTheorem*, *Roots*, *Rule*, *Show*, *ShowIncomplete*, *ShowSolution*, *ShowSteps*, *Summand*, *SurfaceOfRevolution*, *SurfaceOfRevolutionTutor*, *Tangent*, *TangentSecantTutor*, *TangentTutor*, *TaylorApproximation*, *TaylorApproximationTutor*, *Understand*, *Undo*, *VolumeOfRevolution*, *VolumeOfRevolutionTutor*, *WhatProblem*] **(1.5)**

Eksamensoppgave 2006 vår / 5

a) Gitt F og CurlF, finn f(y,z) når f(0,0) = 1

F

$F := (x, y, z) \to \langle f(y, z), 2 \cdot y \cdot z, g(y, z) \rangle;$

$$(x, y, z) \to Student:\text{-}VectorCalculus:\text{-}<,>(f(y, z), 2\,y\,z, g(y, z)) \quad \textbf{(2.1.1)}$$

Curl F

$Curl(F) = \langle h(y, z), 2\,yz, -z^2 \rangle;$

$$\left((x, y, z) \to VectorCalculus:\text{-}Vector\left(\left[\frac{\partial}{\partial y} g(y, z) - 2\,y, \frac{\partial}{\partial z} f(y, z), -\left(\frac{\partial}{\partial y} f(y, z)\right)\right], attributes = [vectorfield, coords = cartesian_{x, y, z}]\right)\right) = (h(y, z))e_x + 2\,yze_y - z^2 e_z \quad \textbf{(2.1.2)}$$

f(y,z) = ... + C(y)

$int(2 \cdot y \cdot z, z);$

$$y\,z^2 \quad \textbf{(2.1.3)}$$

f(y,z) = ... + C(z)

$int(z^2, y);$

$$y\,z^2 \quad \textbf{(2.1.4)}$$

f(y,z) = y z^2 + 1

b) Gitt S, regn ut Curl integralet

S er en del av en paraboloide

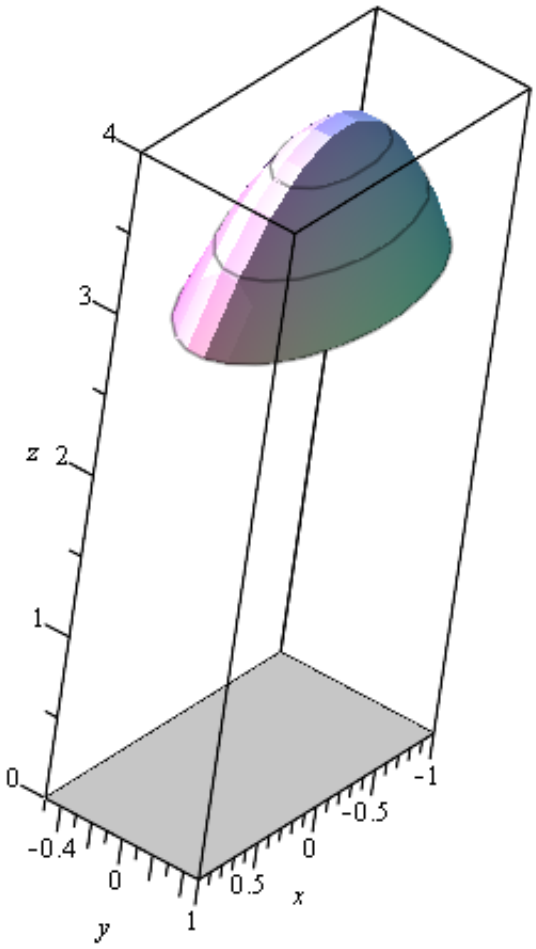
$$S := implicitplot3d\left(z = 4 - x^2 - 4 \cdot y^2,\ x = -1..1,\ y = -\frac{1}{2}..\frac{1}{2},\ z = 3..4,\ style = surfacecontour,\ axes = boxed,\ scaling = constrained\right);$$

$$XYplanet := plot3d\left([x, y, 0],\ x = -1..1,\ y = -\frac{1}{2}..\frac{1}{2},\ color = gray,\ style = surface\right);$$

$$display(S, XYplanet);$$

$$PLOT3D(\ldots)$$

$$PLOT3D(\ldots)$$



Stokes teorem -> det er nok å finne sirkulasjonen på randkurven

Randkurven er en ellipse

$r := t \to \left\langle \cos(t), \frac{1}{2} \cdot \sin(t), 3 \right\rangle;$

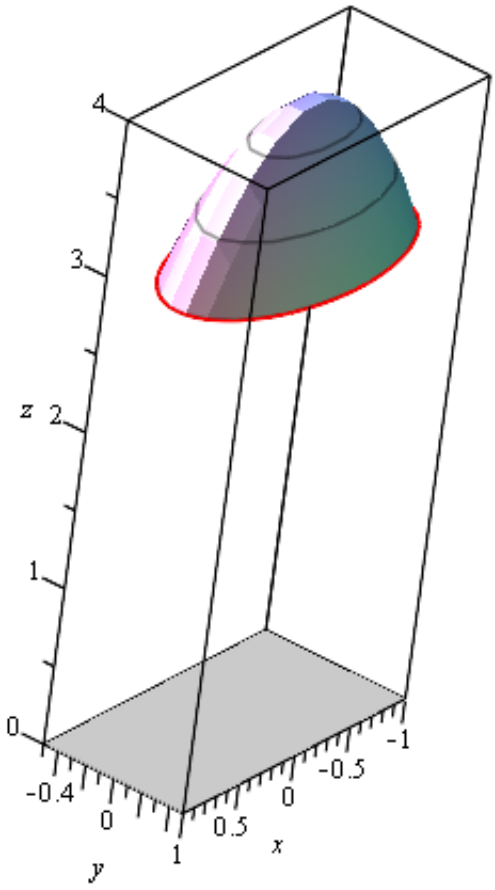
$$t \to Student:\text{-}VectorCalculus:\text{-}<,>\left(\cos(t), 1\ \frac{1}{2}\ \sin(t), 3\right) \qquad \textbf{(2.2.1)}$$

Se på bildet

$C := SpaceCurve(r(t), t = 0..2 \cdot \mathrm{Pi}, color = red);$

$display(S, XYplanet, C);$

$$PLOT3D(\ldots)$$



Graph of the curve represented parametrically by the components of the given vector.

Tangentvektoren

$T := t \rightarrow TangentVector(r(t));$

$t \rightarrow Student:\text{-}VectorCalculus:\text{-}TangentVector(r(t))$ **(2.2.2)**

$T(t);$

$$\begin{bmatrix} -\sin(t) \\ \frac{1}{2}\cos(t) \\ 0 \end{bmatrix} \quad \textbf{(2.2.3)}$$

$convert(r(t), list);$

$$\left[\cos(t), \frac{1}{2}\sin(t), 3\right] \quad \textbf{(2.2.4)}$$

F langs kurven

$F\left(\cos(t), \frac{1}{2}\sin(t), 3\right);$

$$\left(f\left(\frac{1}{2}\sin(t), 3\right)\right)e_x + 3\sin(t)e_y + \left(g\left(\frac{1}{2}\sin(t), 3\right)\right)e_z \quad \textbf{(2.2.5)}$$

F * T

$DotProduct\left(F\left(\cos(t), \frac{1}{2}\cdot\sin(t), 3\right), T(t)\right);$

$$-f\left(\frac{1}{2}\sin(t), 3\right)\sin(t) + \frac{3}{2}\sin(t)\cos(t) \quad \textbf{(2.2.6)}$$

f(y,z) fra del a)

$f := (y, z) \to y\cdot z^2 + 1;$

$$(y, z) \to y z^2 + 1 \quad \textbf{(2.2.7)}$$

Integralet

$int\left(DotProduct\left(F\left(\cos(t), \frac{1}{2}\cdot\sin(t), 3\right), T(t)\right), t = 0..2\cdot\mathrm{Pi}\right);$

$$-\frac{9}{2}\pi \quad \textbf{(2.2.8)}$$

Eksempel 2

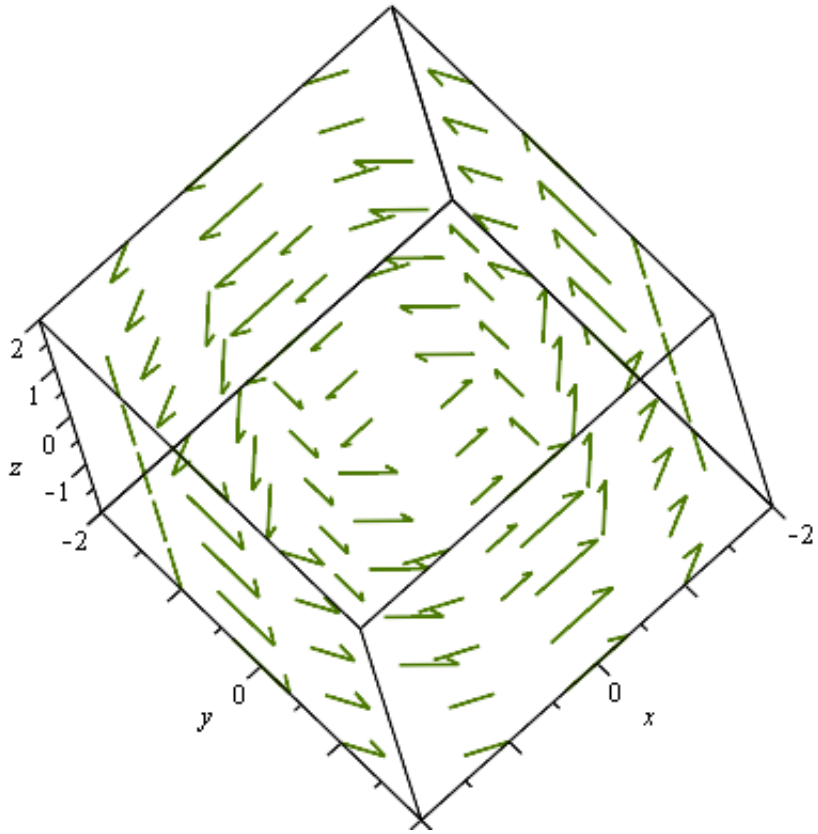
Rotasjon med vinkel Pi / 2 og vinkelhastighet a om z-aksen (a = 1 --> vinkelhastighet 1)

$F := (x, y, z) \to \langle -a\cdot y, a\cdot x, 0\rangle;$

$$(x, y, z) \to Student:\text{-}VectorCalculus:\text{-}<,>(Student:\text{-}VectorCalculus:\text{-}`-`(a\,y), a\,x, 0) \quad \textbf{(3.1)}$$

Rotasjonsfeltet (a = 1)

$VectorField(subs(a = 1, F(x, y, z)), output = plot, axes = boxed);$



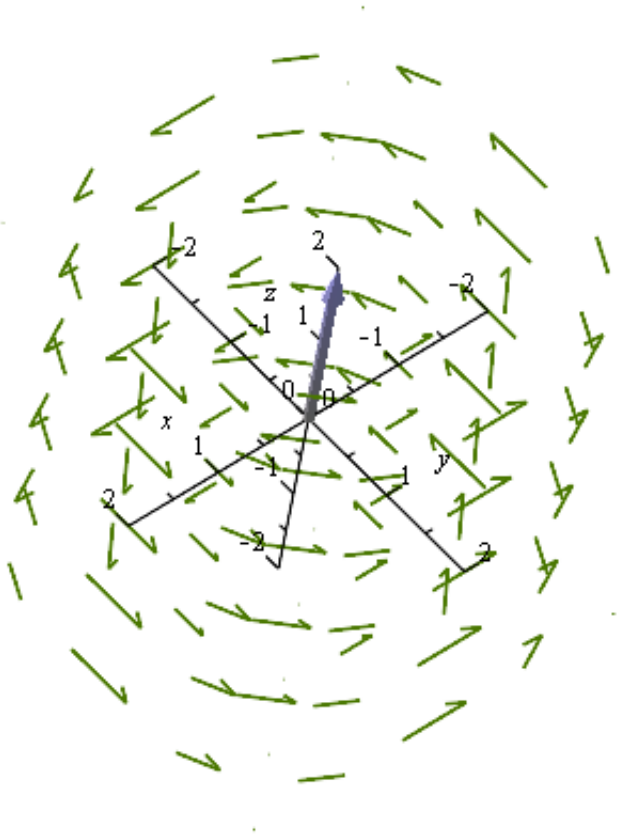
Curl

$Curl(F)$;

$(x, y, z) \rightarrow VectorCalculus:\text{-}Vector([0, 0, 2\,a], attributes = [vectorfield, coords = cartesian_{x, y, z}])$ **(3.2)**

Curlvektoren er parallel med rotasjonsakse

$display(VectorField(subs(a = 1, F(x, y, z))), output = plot, axes = normal), arrow(subs(a = 1, Curl(F)(x, y, z))))$;



Størrelsen til Curlvektoren er 2 * vinkelhastighet

$Norm(Curl(F)(x, y, z))$;

$$2\sqrt{a^2} \tag{3.3}$$