> $with(Student[MultivariateCalculus]):$

> $f := (x, y) \rightarrow \exp\left(\frac{x-y}{x+y}\right):$

> $IntegralXY := Int(Int(f(x, y), y), x)$

$$IntegralXY := \iint e^{\frac{x-y}{x+y}}\, \mathrm{d}y\, \mathrm{d}x \tag{1}$$

> $IntegralUV := ChangeOfVariables\left(IntegralXY, \left[x = \frac{1}{2}(u+v), y = \frac{1}{2}(-u+v)\right]\right)$

$$IntegralUV := \iint \frac{1}{2}\, e^{\frac{u}{v}}\, \mathrm{d}u\, \mathrm{d}v \tag{2}$$

> $MultiInt\left(\frac{1}{2}\cdot\exp\left(\frac{u}{v}\right), u=-v..v, v=0..1, output=steps\right)$

$$\int_0^1 \int_{-v}^{v} \frac{e^{\frac{u}{v}}}{2}\, \mathrm{d}u\, \mathrm{d}v$$

$$= \int_0^1 \left(\left. \frac{v\, e^{\frac{u}{v}}}{2} \right|_{u = -v..v} \right) \mathrm{d}v$$

$$= \int_0^1 \left(-\frac{v\, e^{-1}}{2} + \frac{v\, e}{2} \right) \mathrm{d}v$$

$$= \left. \left(-\frac{v^2\, e^{-1}}{4} + \frac{v^2\, e}{4} \right) \right|_{v=0..1}$$

$$-\frac{1}{4}\, e^{-1} + \frac{1}{4}\, e \tag{3}$$

> $MultiInt(f(x, y), x = 0..1 - y, y = 0..1, output = steps)$

$$\int_0^1 \int_0^{1-y} e^{\frac{x-y}{x+y}} \, dx \, dy$$

$$= \int_0^1 \left(\left. \left(2y \left(\frac{e^{1-\frac{2y}{x+y}} (x+y)}{2y} - e \, \mathrm{Ei}_1\left(\frac{2y}{x+y} \right) \right) \right) \right|_{x=0..1-y} \right) dy$$

$$= \int_0^1 \int_0^{1-y} e^{\frac{x-y}{x+y}} \, dx \, dy$$

$$= \left. \int \int_0^{1-y} e^{\frac{x-y}{x+y}} \, dx \, dy \right|_{y=0..1}$$

$$-\frac{1}{4} e^{-1} + \frac{1}{4} e \qquad \textbf{(4)}$$

> $T := (u, v) \to \left(\frac{1}{2} (u+v), \frac{1}{2} (-u+v) \right)$:

> $J := (u, v) \to Jacobian([T(u, v)[1], T(u, v)[2]], [u, v], output = determinant)$:

> $g := (u, v) \to (f@T)(u, v)$:

> $MultiInt(g(u, v) \cdot \mathrm{abs}(J(u, v)), u = -v..v, v = 0..1, output = steps)$

$$\int_0^1 \int_{-v}^{v} \frac{e^{\frac{u}{v}}}{2} \, du \, dv$$

$$= \int_0^1 \left(\left. \frac{v \, e^{\frac{u}{v}}}{2} \right|_{u=-v..v} \right) dv$$

$$= \int_0^1 \left(-\frac{v \, e^{-1}}{2} + \frac{v \, e}{2} \right) dv$$

$$= \left. \left(-\frac{v^2 e^{-1}}{4} + \frac{v^2 e}{4} \right) \right|_{v=0..1}$$

$$-\frac{1}{4} e^{-1} + \frac{1}{4} e \qquad \textbf{(5)}$$

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