```
> with(plots):
> with(Student[MultivariateCalculus]):
> MultiInt(1, z = 0..1 - y, y = x^2..1, x = -1..1, output = steps)
```

$$\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} 1 \, dz \, dy \, dx$$

$$= \int_{-1}^{1}\int_{x^2}^{1} \left(z\Big|_{z=0..1-y} \right) dy \, dx$$

$$= \int_{-1}^{1}\int_{x^2}^{1} (1-y) \, dy \, dx$$

$$= \int_{-1}^{1} \left(\left(y - \frac{1}{2} y^2 \right)\Big|_{y=x^2..1} \right) dx$$

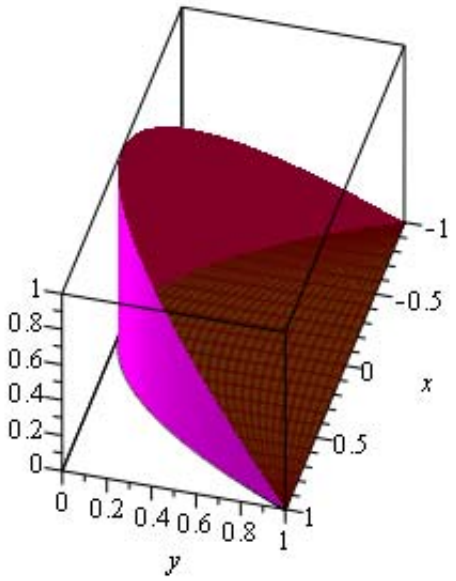
$$= \int_{-1}^{1} \left(\frac{1}{2} - x^2 + \frac{1}{2} x^4 \right) dx$$

$$= \left(\frac{1}{2} x - \frac{1}{3} x^3 + \frac{1}{10} x^5 \right)\Big|_{x=-1..1}$$

$$\frac{8}{15} \qquad \textbf{(1)}$$

```
> T1oppe := plot3d([x, y, 1 - y], x = -1..1, y = x^2..1, color = "Red", style = patchnogrid,
      transparency = 0.3):
> T1nede := plot3d([x, y, 0], x = -1..1, y = x^2..1, color = "Green"):
> T1side := plot3d([x, x^2, 1 - y], x = -1..1, y = x^2..1, color = "Magenta", style = patchnogrid):
> display(T1oppe, T1nede, T1side, scaling = constrained, axes = boxed, orientation = [15, 50], title
      = "Integrasjonsområdet til (1)")
```

Integrasjonområdet til (1)



> $MultiInt\left(1, z = 3\cdot x^2 + 3\cdot y^2 - 16 \,..\, 9 - x^2 - y^2, y = -\mathrm{sqrt}\left(\frac{25}{4} - x^2\right) \,..\, \mathrm{sqrt}\left(\frac{25}{4} - x^2\right), x = -\frac{5}{2} \,..\, \frac{5}{2}, output = steps\right)$

$$\int_{-\frac{5}{2}}^{\frac{5}{2}} \int_{-\frac{\sqrt{25-4x^2}}{2}}^{\frac{\sqrt{25-4x^2}}{2}} \int_{3x^2+3y^2-16}^{9-x^2-y^2} 1 \, dz \, dy \, dx$$

$$= \int_{-\frac{5}{2}}^{\frac{5}{2}} \int_{-\frac{\sqrt{25-4x^2}}{2}}^{\frac{\sqrt{25-4x^2}}{2}} \left(z \Big| z = 3x^2 + 3y^2 - 16 \,..\, 9 - x^2 - y^2 \right) dy \, dx$$

$$= \int_{-\frac{5}{2}}^{\frac{5}{2}} \int_{-\frac{\sqrt{25-4x^2}}{2}}^{\frac{\sqrt{25-4x^2}}{2}} \left(25 - 4x^2 - 4y^2 \right) dy \, dx$$

$$= \int_{-\frac{5}{2}}^{\frac{5}{2}} \left(\left(25y - 4x^2 y - \frac{4}{3} y^3 \right) \Big| y = -\frac{\sqrt{25-4x^2}}{2} \,..\, \frac{\sqrt{25-4x^2}}{2} \right) dx$$

$$= \int_{-\frac{5}{2}}^{\frac{5}{2}} \left(25\sqrt{25-4x^2} - 4x^2\sqrt{25-4x^2} - \frac{\left(25-4x^2\right)^{3/2}}{3} \right) dx$$

$$= \left(\frac{25x\sqrt{25-4x^2}}{4} + \frac{625 \arcsin\left(\frac{2x}{5}\right)}{8} + \frac{x\left(25-4x^2\right)^{3/2}}{6} \right) \Bigg| x = -\frac{5}{2} \,..\, \frac{5}{2}$$

$$\frac{625}{8}\pi \qquad \textbf{(2)}$$

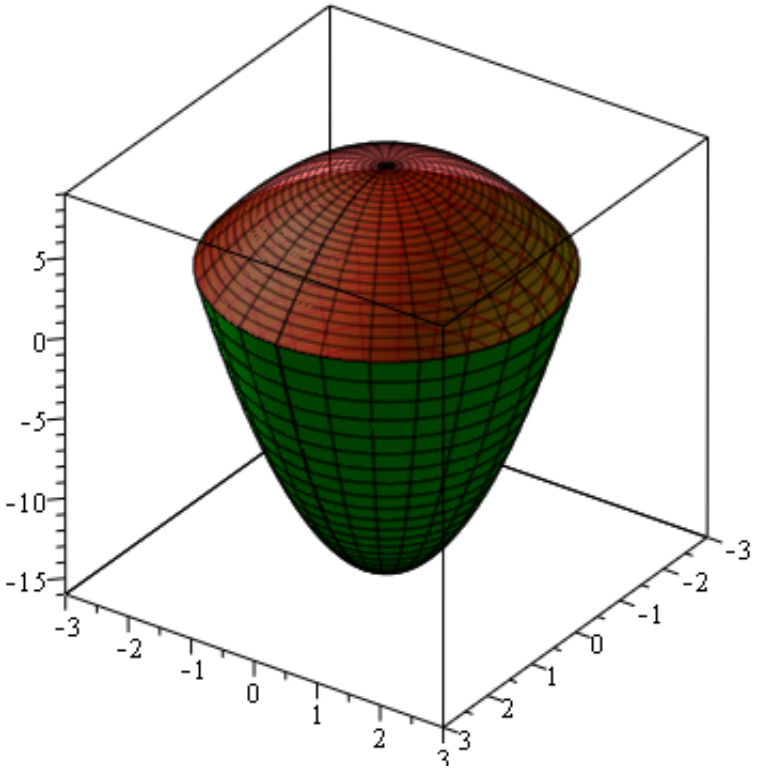
> $T2ovre := plot3d\left([r, \text{theta}, 9 - r^2], r = 0 \,..\, \frac{5}{2}, \text{theta} = 0 \,..\, 2 \cdot \text{Pi}, coords = cylindrical, color \right.$

= "Red", *transparency* = 0.4) :

> $T2nedre := plot3d\left([r, \text{theta}, 3 \cdot r^2 - 16], r = 0 .. \frac{5}{2}, \text{theta} = 0 .. 2 \cdot \text{Pi}, coords = cylindrical, color = \text{"Green"} \right) :$

> $display(T2ovre, T2nedre, view = [-3..3, -3..3, -16..9], axes = boxed, orientation = [35, 60], title = \text{"Integrasjonsområdet til (2)"})$

Integrasjonsområdet til (2)



> $MultiInt(\exp(x^3), y = 0 .. x, x = z .. 1, z = 0 .. 1, output = steps)$

$$\int_0^1 \int_z^1 \int_0^x e^{x^3}\, dy\, dx\, dz$$

$$= \int_0^1 \int_z^1 \left(e^{x^3} y \Big|_{y=0..x} \right) dx\, dz$$

$$= \int_0^1 \int_z^1 e^{x^3} x\, dx\, dz$$

$$= \int_0^1 \left(-\frac{(-1)^{1/3} \left(\frac{x^2 (-1)^{2/3} \Gamma\left(\frac{2}{3}\right)}{(-x^3)^{2/3}} - \frac{x^2 (-1)^{2/3} \Gamma\left(\frac{2}{3}, -x^3\right)}{(-x^3)^{2/3}} \right)}{3} \Bigg|_{x=z..1} \right) dz$$

$$= \int_0^1 \int_z^1 e^{x^3} x\, dx\, dz$$

$$= \left(z \left(\int_z^1 e^{x^3} x\, dx \right) + \frac{e^{z^3}}{3} \right) \Big|_{z=0..1}$$

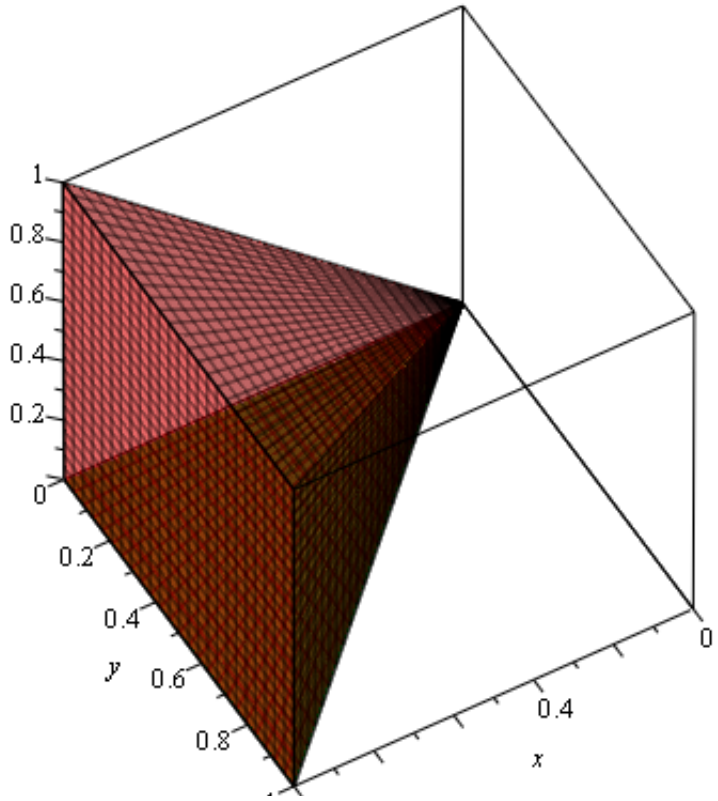
$$\int_0^1 \int_z^1 e^{x^3} x\, dx\, dz \qquad \textbf{(3)}$$

```
> evalf(%)
```

$$0.5727606094 \qquad \textbf{(4)}$$

```
> T3oppe := plot3d([x, y, x], x = 0..1, y = 0..x, color = "Red", transparency = 0.4):
> T3nede := plot3d([x, y, 0], x = 0..1, y = 0..x, color = "Green"):
> T3fremme := plot3d([1, y, z], y = 0..1, z = 0..1, color = "Red", transparency = 0.4):
> T3venstreside := plot3d([x, 0, z], x = 0..1, z = 0..x, color = "Red", transparency = 0.4):
> T3hoyreside := plot3d([x, x, z], x = 0..1, z = 0..x, color = "Red", transparency = 0.4):
> display(T3oppe, T3nede, T3fremme, T3venstreside, T3hoyreside, scaling = constrained, axes
      = boxed, orientation = [60, 40], title = "Integrasjonsområdet til (3) og (5)")
```

Integrasjonsområdet til (3) og (5)



> $MultiInt(\exp(x^3), z = 0..x, y = 0..x, x = 0..1, output = steps)$

$$\int_0^1 \int_0^x \int_0^x e^{x^3}\, dz\, dy\, dx$$

$$= \int_0^1 \int_0^x \left(e^{x^3} z \Big|_{z=0..x} \right) dy\, dx$$

$$= \int_0^1 \int_0^x e^{x^3} x\, dy\, dx$$

$$= \int_0^1 \left(e^{x^3} x y \Big|_{y=0..x} \right) dx$$

$$= \int_0^1 e^{x^3} x^2\, dx$$

$$= \frac{e^{x^3}}{3}\Big|_{x=0..1}$$

$$-\frac{1}{3} + \frac{1}{3} e \qquad \textbf{(5)}$$

```
> evalf(%)
```

$$0.5727606093 \qquad \textbf{(6)}$$

```
>
```