Cramers regel

System $\quad A\underline{x} = \underline{b}$

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \qquad \underline{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \qquad \underline{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

Hvis $\det A \neq 0$, så er løsningen

$$x_i = \frac{1}{\det A} \begin{vmatrix} a_{11} & \cdots & b_1 & \cdots & a_{1n} \\ \vdots & & \vdots & & \\ a_{n1} & \cdots & b_n & \cdots & a_{nn} \end{vmatrix}$$

kolonne i

Eks

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \qquad \underline{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Her er $\det A = 5$ (sjekk dette)

Løsningen av $A\underline{x} = \underline{b}$ er

$$x_1 = \frac{1}{5} \begin{vmatrix} 1 & 0 & 1 \\ 0 & 3 & 4 \\ 1 & 1 & 2 \end{vmatrix} = \frac{1}{5}\left(1 \cdot \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix} + 1 \cdot \begin{vmatrix} 0 & 3 \\ 1 & 1 \end{vmatrix}\right) = -\frac{1}{5}$$

$$x_2 = \frac{1}{5} \begin{vmatrix} 2 & 1 & 1 \\ 1 & 0 & 4 \\ 0 & 1 & 2 \end{vmatrix} = \frac{1}{5}\left(-1 \cdot \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} - 1 \cdot \begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix}\right) = -\frac{9}{5}$$

Beregnet langs
2. kolonne

$$x_3 = \frac{1}{5} \begin{vmatrix} 2 & 0 & 1 \\ 1 & 3 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \frac{1}{5}\left(2 \cdot \begin{vmatrix} 3 & 0 \\ 1 & 1 \end{vmatrix} + 1 \cdot \begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix}\right) = \frac{7}{5}$$