Eks
$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

Diagonaliser A hvis mulig

Løsning

Kar. ligning:
$$\det(A-\lambda I) = \begin{vmatrix} -\lambda & 0 & -2 \\ 1 & 2-\lambda & 1 \\ 1 & 0 & 3-\lambda \end{vmatrix}$$

$$= -\lambda \begin{vmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{vmatrix} + (-2) \begin{vmatrix} 1 & 2-\lambda \\ 1 & 0 \end{vmatrix}$$

$$= -\lambda^3 + 5\lambda^2 - 8\lambda + 4 = 0$$

Mulige heltallsløsninger:

$-4, -2, -1, 1, 2, 4$

$1, 2$ passer inn ($\lambda = 2$ dobbel rot)

Egenverdier : $\lambda = 1$, $\lambda = 2$

Må finne egenvektorene for hver egenverdi

$$(A - \lambda I)\underline{v} = \underline{0}$$

$\underline{\lambda = 1}$:
$$\begin{bmatrix} -1 & 0 & -2 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\xrightarrow{\text{radop.}} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{aligned} x_3 &= t \\ x_2 &= t \\ x_1 &= -2t \end{aligned}$$

Egenvektor : $\underline{v} = t \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$ Basis : $\left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \right\}$

$\underline{\lambda = 2}$:
$$\begin{bmatrix} -2 & 0 & -2 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \xrightarrow{\text{radop}} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_2 = s, \; x_3 = t, \; x_1 = -t$$

Egenvektor :

$$\underline{v} = s \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$ Basis : $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$