Gram-Schmidt

La V være underrommet av $\mathbb{R}^4$ med basis

$$\left\{ \underline{v_1} = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \underline{v_2} = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \underline{v_3} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix} \right\}$$

Finn en ortogonal basis til V.

Løsning

Velger $\underline{u_1} = \underline{v_1} = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}$

$$\underline{u_2} = \underline{v_2} - \frac{\underline{u_1} \cdot \underline{v_2}}{\underline{u_1} \cdot \underline{u_1}} \underline{u_1} = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \end{bmatrix} - \frac{8}{10} \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{2}{5} \\ \frac{6}{5} \\ -\frac{3}{5} \\ -\frac{4}{5} \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 2 \\ 6 \\ -3 \\ -4 \end{bmatrix}$$

Siden det er retningen til $\underline{u_2}$ som er viktig og ikke størrelsen, så kan vi stryke faktoren $\frac{1}{5}$ og velge

$$\underline{u_2} = \begin{bmatrix} 2 \\ 6 \\ -3 \\ -4 \end{bmatrix}$$

$$\underline{u_3} = \underline{v_3} - \frac{\underline{u_1} \cdot \underline{v_3}}{\underline{u_1} \cdot \underline{u_1}} \underline{u_1} - \frac{\underline{u_2} \cdot \underline{v_3}}{\underline{u_2} \cdot \underline{u_2}} \underline{u_2}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix} - \frac{6}{10} \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} - \frac{11}{65} \begin{bmatrix} 2 \\ 6 \\ -3 \\ -4 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{70}{130} \\ \frac{50}{130} \\ \frac{40}{130} \\ \frac{10}{130} \end{bmatrix} = \frac{1}{13} \begin{bmatrix} -7 \\ 5 \\ 4 \\ 1 \end{bmatrix}$$

Kan velge $\underline{u_3} = \begin{bmatrix} -7 \\ 5 \\ 4 \\ 1 \end{bmatrix}$

Ortogonal basis for V:

$$\left\{ \underline{u_1} = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \underline{u_2} = \begin{bmatrix} 2 \\ 6 \\ -3 \\ -4 \end{bmatrix}, \underline{u_3} = \begin{bmatrix} -7 \\ 5 \\ 4 \\ 1 \end{bmatrix} \right\}$$