Eks

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

Kar. Ligning:

$$\det(A-\lambda I) = \begin{vmatrix} 3-\lambda & 2 & 2 \\ 2 & 3-\lambda & 2 \\ 2 & 2 & 3-\lambda \end{vmatrix}$$

$$= -\lambda^3 + 9\lambda^2 - 15\lambda + 7 = 0$$

Må løse $\lambda^3 - 9\lambda^2 + 15\lambda - 7 = 0$

Mulige heltallsløsninger: $-7, -1, 1, 7$

Får etter regning:

$$\lambda^3 - 9\lambda^2 + 15\lambda - 7 = (\lambda-1)^2(\lambda-7)$$

Egenverdier: $\lambda = 1$ (dobbel rot)
$\lambda = 7$

Egenvektorer: $(A-\lambda I)\underline{v} = \underline{0}$

$\underline{\lambda = 1}$:

$$\begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\xrightarrow{\text{radop}} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{aligned} x_2 &= s \\ x_3 &= t \\ x_1 &= -s-t \end{aligned}$$

$$\underline{v} = s \underbrace{\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}}_{= \underline{v_1}} + t \underbrace{\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}}_{= \underline{v_2}}$$

Basis: $\{\underline{v_1}, \underline{v_2}\}$

Bruker Gram-Schmidt på denne

$$\underline{u_1} = \underline{v_1} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$\underline{u_2} = \underline{v_2} - \frac{\underline{u_1} \cdot \underline{v_2}}{\underline{u_1} \cdot \underline{u_1}} \underline{u_1} = \underline{v_2} - \frac{1}{2} \underline{u_1} = \begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{bmatrix}$$