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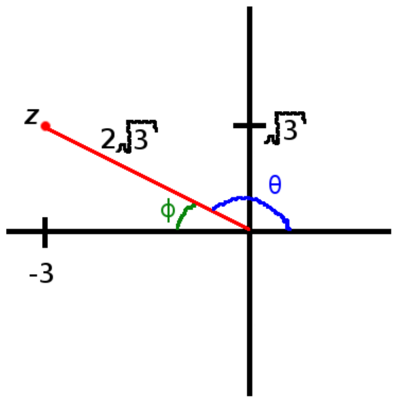
Solutions to exercise set 1

1 Given $z = -3 + \sqrt{3}i$, we are supposed to find $|z|$ and $\text{Arg}(z)$. We have

$$|z| = \sqrt{(-3)^2 + (\sqrt{3})^2} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3}$$

To find the argument, it is convenient to draw a simple figure:

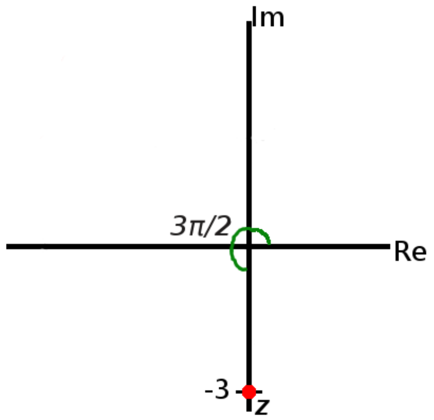


The principal argument is the angle θ, so we can for instance use our favourite trigonometric function to find ϕ and then compute $\theta = \pi - \phi$. Using cosine, we get

$$\cos\phi = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$
$$\phi = \cos^{-1}(\frac{\sqrt{3}}{2}) = \frac{\pi}{6}$$
$$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

So $\text{Arg}(z) = \frac{5\pi}{6}$.

2 We have z with $|z| = 3$ and $\arg(z) = \frac{3\pi}{2}$. We want to find x and y such that $z = x + iy$. Again, we draw a figure:



We can easily see that $z = -3i$. (This means that $x = 0$ and $y = -3$.)

3 Simplifying the expression can be done in the following way:

$$\frac{3+i}{1-i} - 2i = \frac{(3+i)(1+i)}{(1-i)(1+i)} - 2i = \frac{2+4i}{2} - 2i = 1 + 2i - 2i = 1$$

The first (and only notable) step was to expand the fraction with $\overline{1-i} = 1+i$.

4 We are supposed to find the three complex numbers w_1, w_2 and w_3 such that $w_i^3 = -27$. We know that $(-3)^3 = -27$, so then we already have one root, say $w_1 = -3$. We find the next root by rotating w_1 by $\frac{2\pi}{3}$. Polar form is convenient here, because we known that $|w_2| = 3$ (all roots have the same modulus), and the argument of w_2 is simply $\pi + \frac{2\pi}{3}$.

$$\begin{aligned} w_2 = 3(\cos(\pi + \frac{2\pi}{3}) + i\sin(\pi + \frac{2\pi}{3})) &= 3(\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3})) \\ &= 3(\frac{1}{2} - i\frac{\sqrt{3}}{2}) = \frac{3}{2}(1 - \sqrt{3}i) \end{aligned}$$

Finally, we know that the third root is the conjugate of w_2 (draw a picture if you don't see this), so $w_3 = \frac{3}{2}(1 + \sqrt{3}i)$.