Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4110, Fall 2012, Test 2, week 36

Exercise group number:

Name:

NTNU username:_____@stud.ntnu.no

You have 20 minutes to solve the 4 problems. Each problem is worth 10 points, and you need at least 20 points to pass the test. Your answers must be written on this sheet of paper (you can use additional sheets if you need more space for your answers). You are allowed to use a Citizen SR-270X or Hewlett Packard HP30S calculator, and Rottman's "Matematisk formelsamling", but no books or notes.

Problem 1 Let D be the set of complex numbers z satisfying $0 \leq \mathrm{Re}(z) \leq \pi/4$ and $-1 \leq \mathrm{Im}(z) \leq 1$. Describe the image R of D under the map $f(z) = e^{2iz}$.

Problem 2 Check that $z_1 = i$ is a zero of the polynomial $P(z) = z^4 - 2z^3 + 3z^2 - 2z + 2$ and find the 3 other zeros of P.

Problem 3 For each of the 3 second-order differential equations

1. $y'' + 3 = 0$
2. $y'' + \sin(2t)y' = 0$
3. $y'' + \sin(y)y = 0$

decide whether the equation is linear or nonlinear. If the equation is linear, state whether the equation is homogeneous or inhomogeneous.

Problem 4 Show that $y_1(t) = e^t$ and $y_2(t) = te^t$ form a fundamental set of solutions for the differential equation $y'' - 2y' + y = 0$, then find a solution y satisfying $y(0) = 1$ and $y'(0) = 0$.