TMA4110 Calculus 3
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Solutions to exercise set 5

Norwegian University of Science and Technology
Department of Mathematical Sciences

1 We have two vectors $\mathbf{a_1}$ and $\mathbf{a_2}$. The task is to determine whether a third vector $\mathbf{b}$ is in Span$\{\mathbf{a_1}, \mathbf{a_2}\}$, that is, to find out if there exists numbers x_1 and x_2 such that $x_1\mathbf{a_1} + x_2\mathbf{a_2} = \mathbf{b}$. One way of solving this is to check if the system $A\mathbf{x} = \mathbf{b}$ has a solution, where $A = [\mathbf{a_1}\ \mathbf{a_2}]$ and $\mathbf{x}$ is the vector $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. We reduce the augmented matrix of the system and get

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

which is inconsistent (the last row says that $0 \cdot x_2 = -1$), so $\mathbf{b}$ is not in Span$\{\mathbf{a_1}, \mathbf{a_2}\}$.

Note: We can see directly from the second row of the first matrix that $x_1 = 1$. The first row then gives $x_2 = 2 - 1 = 1$, but this does not fit into the third row ($3 + 2$ is 5, not 4).

2 We reduce the augmented matrix of $A\mathbf{x} = \mathbf{b}$:

$$\begin{bmatrix} 1 & 2 & -3 & 5 \\ 2 & 1 & -3 & 13 \\ -1 & 1 & 0 & -8 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 7 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

We see that x_3 is a free variable, so we set $x_3 = t$, where t can be any real number. The solution can then be written as

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7+t \\ -1+t \\ t \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

3 Let p_G and p_S be the prices of the total annual output of Goods and Services respectively. Then

$$p_G = 0.6p_S + 0.3p_g$$
$$p_S = 0.7p_g + 0.4p_s$$

which rewrites to

$$0.7p_G - 0.6p_S = 0$$
$$-0.7p_g + 0.6p_S = 0$$

Solving this, we get one free variable p_S, and the equilibrium price vector

$$\mathbf{p} = p_S \begin{bmatrix} \frac{6}{7} \\ 1 \end{bmatrix}$$

We have that $p_G = \$600$, so $p_S = \frac{7}{6} \cdot \$600 = \$700$.