TMA4110, Fall 2012, Test 6, week 40

Exercise group number:

Name:

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You have 20 minutes to solve the 4 problems. Each problem is worth 10 points, and you need at least 20 points to pass the test. Your answers must be written on this sheet of paper (you can use additional sheets if you need more space for your answers). You are allowed to use a Citizen SR-270X or Hewlett Packard HP30S calculator, and Rottman's "Matematisk formelsamling", but no books or notes.

Problem 1   Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$.

Determine whether the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent or linearly dependent.

Problem 2   Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be a linear transformation such that $T(x_1, x_2, x_3, x_4) = (x_1 + 4x_2 - x_3 + 3x_4, x_2 + 2x_3 - x_4, 2x_1 + 9x_2 + 5x_4)$. Find the standard matrix of T.

Problem 3　　Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be a linear transformation such that $T(x_1, x_2, x_3, x_4) = (x_1 + 4x_2 - x_3 + 3x_4, x_2 + 2x_3 - x_4, 2x_1 + 9x_2 + 5x_4)$. Determine if T is onto.

Problem 4　　In a certain region, 10% of a city's population moves to the surrounding suburbs each year, and 20% of the suburban population moves into the city. In 2010, there were 200,000 residents in the city and 100,000 in the suburbs. Find the population in the city in the suburbs two years later, in 2012.