

- 1 We have $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, and the task is to determine whether the set is linearly independent or linearly dependent. It is linearly independent if and only if the equation $a\mathbf{v}_1 + b\mathbf{v}_2 + c\mathbf{v}_3 = \mathbf{0}$, where a, b, c are scalars, has only the trivial solution $a = b = c = 0$.

We reduce the augmented matrix of the vector equation:

$$\begin{bmatrix} 1 & 3 & 5 & 0 \\ 2 & -1 & -4 & 0 \\ -3 & 0 & 3 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

c is a free variable, and the system has infinitely many solutions. For instance, if we chose $c = -1$, we get $a = 1$ and $b = -2$, and we can write \mathbf{v}_3 as $\mathbf{v}_3 = \mathbf{v}_1 - 2\mathbf{v}_2$. The set is linearly dependent.

- 2 The standard matrix of T is of the form $[T(\mathbf{e}_1) \ T(\mathbf{e}_2) \ T(\mathbf{e}_3) \ T(\mathbf{e}_4)]$. We compute

$$T(\mathbf{e}_1) = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \quad T(\mathbf{e}_2) = \begin{bmatrix} 4 \\ 1 \\ 9 \end{bmatrix} \quad T(\mathbf{e}_3) = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} \quad T(\mathbf{e}_4) = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}$$

Hence the standard matrix is

$$A = \begin{bmatrix} 1 & 4 & -1 & 3 \\ 0 & 1 & 2 & -1 \\ 2 & 9 & 0 & 5 \end{bmatrix}$$

This can also be done “by inspection”, as in Example 5 on page 77.

- 3 T is the linear transformation from Problem 2, so we have the standard matrix

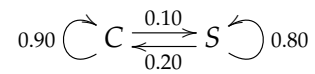
$$A = \begin{bmatrix} 1 & 4 & -1 & 3 \\ 0 & 1 & 2 & -1 \\ 2 & 9 & 0 & 5 \end{bmatrix}$$

According to Theorem 12, page 77, T is onto if and only if the columns of A span \mathbb{R}^3 . This happens if and only if A has a pivot position in each row. We reduce A and get

$$A = \begin{bmatrix} 1 & 4 & -1 & 3 \\ 0 & 1 & 2 & -1 \\ 2 & 9 & 0 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & -1 & 3 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

A does not have a pivot position in the third row, and hence the columns does not span \mathbb{R}^3 . Then T is not onto.

- 4 We draw a picture of the migration (C means city and S means suburbs):



The migration matrix is given by

$$M = \begin{bmatrix} 0.90 & 0.20 \\ 0.10 & 0.80 \end{bmatrix}$$

and the population in 2010 is given by

$$\mathbf{x}_0 = \begin{bmatrix} 200,000 \\ 100,000 \end{bmatrix}$$

We are supposed to find \mathbf{x}_2 , i.e. the population vector for 2012. We have that

$$\mathbf{x}_1 = M\mathbf{x}_0 = \begin{bmatrix} 0.90 & 0.20 \\ 0.10 & 0.80 \end{bmatrix} \cdot \begin{bmatrix} 200,000 \\ 100,000 \end{bmatrix} = \begin{bmatrix} 200,000 \\ 100,000 \end{bmatrix}$$

The number of people living in each region does not change from one year to the next, and hence $\mathbf{x}_2 = \mathbf{x}_1 = \mathbf{x}_0$.

(See pages 84–85 in the book for more details.)