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Solutions to exercise set 7

1 We perform matrix multiplication as described in Chapter 2.1:

$$AB = \begin{bmatrix} 3 & 1 & 0 \\ 0 & -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 4 & 8 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 0 & 0 \end{bmatrix}$$

2 We start by finding the standard matrix for T. By inspection, this is

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 0 & 3 & 2 \end{bmatrix}$$

The standard matrix of T^{-1} is A^{-1}. To find this, we reduce the matrix $[A \quad I]$ to reduced row echelon form. Then the reduced matrix will be $[I \quad A^{-1}]$ (see page 108).

$$\begin{bmatrix} 1 & 0 & -1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 3 & 2 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -1/2 & 3/4 & -1/4 \\ 0 & 1 & 0 & 1 & -1/2 & 1/2 \\ 0 & 0 & 1 & -3/2 & 3/4 & -1/4 \end{bmatrix}$$

Hence, the matrix for A^{-1} is

$$\begin{bmatrix} -1/2 & 3/4 & -1/4 \\ 1 & -1/2 & 1/2 \\ -3/2 & 3/4 & -1/4 \end{bmatrix}$$

and a formula for T^{-1} is

$$T(x_1, x_2, x_3) = \left(-\frac{1}{2}x_1 + \frac{3}{4}x_2 - \frac{1}{4}x_3, x_1 - \frac{1}{2}x_2 + \frac{1}{2}x_3, -\frac{3}{2}x_1 + \frac{3}{4}x_2 - \frac{1}{4}x_3\right)$$

3 We are given 3 matrices, and the task is to determine which are invertible. We know that an invertible matrix M is an $n \times n$-matrix which is row equivalent to the identity matrix I_n. This is the same as M having n pivot positions. With this in mind we inspect the given matrices, performing row operations if necessary.

$$A = \begin{bmatrix} 1 & 5 & 0 & -2 \\ 0 & 3 & 3 & 1 \\ 0 & 0 & -2 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A is already in echelon form, and it is clear that A has 4 pivot position and hence is invertible.

For B, we use the first row to reduce the second and third:

$$B = \begin{bmatrix} 1 & 2 & 0 \\ -1 & -2 & -1 \\ 5 & 10 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

B has only 2 pivot positions and is not invertible. Alternatively, we note that the second column of B is two times the first, which is enough to conclude that B is not invertible (Theorem 4 page 37).

$$C = \begin{bmatrix} -1 & 5 & 2 \\ 2 & -9 & -6 \\ -1 & 6 & 3 \end{bmatrix} \sim \begin{bmatrix} -1 & 5 & 2 \\ 0 & 1 & -2 \\ 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} -1 & 5 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 3 \end{bmatrix}$$

C has three pivot positions and is invertible.