Norwegian University of Science and Technology
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# TMA4110 Calculus 3
# Fall 2012

**Solutions to exercise set 9**

$\boxed{1}$ If $H$ is a subset of $\mathbb{R}^3$, then the three conditions of the definition on page 193 must hold. The first condition is: "The zero vector of $\mathbb{R}^3$ is in $H$." In this case, *any* vector of $H$ has $-2$ as its second coordinate, and hence the zero vector is not in $H$. We conclude that $H$ is not a subspace of $\mathbb{R}^3$.

$\boxed{2}$ We have the matrix $A$, and the task is to find a basis for the null space of $A$. This is achieved by solving the system $A\mathbf{x} = \mathbf{0}$.

$$[A \;\; \mathbf{0}] = \begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -4 & 3 & 0 \\ 0 & 1 & 2 & 0 & 0 \end{bmatrix}$$

We get $x_1 = 4x_3 - 3x_4$ and $x_2 = -2x_3$, and $x_3$ and $x_4$ are free. A general solution is

$$x = \begin{bmatrix} 4x_3 - 3x_4 \\ -2x_3 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} 4 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -3 \\ 0 \\ 0 \\ 1 \end{bmatrix} = x_3\mathbf{u} + x_4\mathbf{v}$$

The basis for the null space of $A$ is $\{\mathbf{u}, \mathbf{v}\}$.

$\boxed{3}$ This time we want to find a basis for the column space of a matrix $A$. By Theorem 6, page 212, the pivot columns of $A$ form a basis for $\mathrm{Col}(A)$. To find the pivots, we reduce $A$ to echelon form:

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & 4 \\ 0 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

The pivot columns are the first and the second, so $\left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \right\}$ is a basis for the column space of $A$.

$\boxed{4}$ We are supposed to find a basis for $\mathrm{Span}\{\mathbf{p_1}, \mathbf{p_2}, \mathbf{p_3}\}$, where $\mathbf{p_1} = 2$, $\mathbf{p_2} = t^2 - 1$ and $\mathbf{p_3} = 2t^2$. It is obvious that $\mathbf{p_2} = \frac{1}{2}(\mathbf{p_3} - \mathbf{p_1})$, so by Theorem 5 page 210, $\mathrm{Span}\{\mathbf{p_1}, \mathbf{p_2}, \mathbf{p_3}\} = \mathrm{Span}\{\mathbf{p_1}, \mathbf{p_3}\}$. It is also clear that $\mathbf{p_1}$ and $\mathbf{p_3}$ are linearly independent. Hence, the set $\{\mathbf{p_1}, \mathbf{p_3}\}$ is a basis for $\mathrm{Span}\{\mathbf{p_1}, \mathbf{p_2}, \mathbf{p_3}\}$.