TMA4110 Calculus 3
Fall 2016

Øving 2

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1.3.1. Let $z_1 = 2 - i$ and $z_2 = 1 + i$. Use the parallelogram law to construct each of the following vectors.

$(a)\quad z_1 + z_2 \qquad (b)\quad z_1 - z_2 \qquad (c)\quad 2z_1 - 3z_2$

1.3.5. Find the following.

$(a)\quad \left|\frac{1+2i}{-2-i}\right| \qquad (b)\quad |(\overline{1+i})(2-3i)(4i-3)| \qquad (c)\quad \left|\frac{i(2+i)^3}{(1-i)^2}\right|$

$(d)\quad \left|\frac{(\pi+i)^{100}}{(\pi-i)^{100}}\right|$

1.3.7. Find the argument of each of the following complex numbers and write each in polar form.

$(a)\quad -\frac{1}{2} \qquad (b)\quad -3+3i \qquad (c)\quad -\pi i \qquad (d)\quad -2\sqrt{3}-2i$

$(e)\quad (1-i)(-\sqrt{3}+i) \qquad (f)\quad (\sqrt{3}-i)^2 \qquad (g)\quad \frac{-1+\sqrt{3}i}{2+2i}$

$(h)\quad \frac{-\sqrt{7}(1+i)}{\sqrt{3}+i}$

1.3.13. Decide which of the following statements are always true.

(a) Arg $z_1 z_2 =$ Arg $z_1 +$ Arg z_2, if $z_1 \neq 0$, $z_2 \neq 0$.
(b) Arg $\bar{z} = -$Arg z, if z is not a real number.
(c) Arg $(z_1/z_2) =$ Arg $z_1 -$ Arg z_2, if $z_1 \neq 0$, $z_2 \neq 0$.
(d) $\arg z =$ Arg $z + 2\pi k$, for $k = 0, \pm 1, \pm 2, \ldots$, if $z \neq 0$.

1.4.1. Write each of the given numbers in the form $a + bi$.

$(a)\quad e^{-i\pi/4} \qquad (b)\quad \frac{e^{1+3i\pi}}{e^{-1+i\pi/2}} \qquad (c)\quad e^{e^i}$

1.4.3. Write each of the given numbers in the polar form $re^{i\theta}$.

$(a)\quad \frac{1-i}{3} \qquad (b)\quad -8\pi(1+\sqrt{3}i) \qquad (c)\quad (1+i)^6$

1.4.7. Show that $e^z = e^{z+2\pi i}$ for all z. (*The exponential function is periodic with period* $2\pi i$.)

1.5.5. Find all the roots of the following.

(a) $(-16)^{1/4}$	(b) $1^{1/5}$	(c) $i^{1/4}$
(d) $(1-\sqrt{3}i)^{1/3}$	(e) $(i-1)^{1/2}$	(f) $\left(\dfrac{2i}{1+i}\right)^{1/6}$

1.5.7. Solve the following equation.

(c) $z^2 - 2z + i = 0$

1.5.9. Solve the equation $z^3 - 3z^2 + 6z - 4 = 0$.