Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4115 MATEMATIKK 3
Midterm (ungraded)

Problem 1 How many solutions of the equation $z^5 = 1 - i$ have positive imaginary part?

A: 1 B: 2 C: 3 D: 5

Problem 2 Consider the equation

$$y'' + 25y = 2x \cos 5x.$$

Which of the following families of functions contains a particular solution of this ODE?

A: $A_0 \cos 5x + B_0 \sin 5x$, ($A_0, B_0$ not both zero)

B: $(A_1 x + A_0) \cos 5x + (B_1 x + B_0) \sin 5x$, ($A_1, B_1$ not both zero)

C: $(A_2 x^2 + A_1 x + A_0) \cos 5x + (B_2 x^2 + B_1 x + B_0) \sin 5x$, ($A_2, B_2$ not both zero)

D: $(A_3 x^3 + A_2 x^2 + A_1 x + A_0) \cos 5x + (B_3 x^3 + B_2 x^2 + B_1 x + B_0) \sin 5x$, ($A_3, B_3$ not both zero)

Problem 3 For which value of k will the solutions of the equation

$$y'' + ky' + 16y = 0$$

have infinitely many zeros?

A: $k > 8$ B: $k < 8$ C: $k > -8$ D: $-8 < k < 8$

Problem 4 Let $y_1(x), y_2(x)$ be solutions of $y'' - 2y' + 2y = 0$ with Wronski determinant $W(y_1, y_2) = W(x)$. If $W(0) = 2$, what is $W(1)$?

A: $2e^2$ B: $3e^2$ C: $2e^{-2}$ D: $3e^{-2}$

Problem 5 For which a does the linear system

$$\begin{aligned} x_1 + \quad x_2 + 2x_3 &= a \\ x_1 + \ 2x_2 + 3x_3 &= 1 \\ 2ax_2 + 4x_3 &= a^2 \end{aligned}$$

have infinitely many solutions?

A: None B: $a = 2$ and $a = 0$ C: $a = 2$ D: $a = 0$

Problem 6 Let $A = \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 6 \\ 8 & 7 \end{bmatrix}$, what is $(AB)^{-1}$?

A: $\begin{bmatrix} -39 & -10 \\ -20 & -9 \end{bmatrix}$ B: $\begin{bmatrix} 9 & -8 \\ 10 & 9 \end{bmatrix}$ C: $\begin{bmatrix} 5 & 4 \\ -6 & -5 \end{bmatrix}$

D: $\begin{bmatrix} -5 & -4 \\ 6 & 5 \end{bmatrix}$