12.19The invertible matrix theorem

The following conditions are equivalent for an $n \times n$-matrix A:

(a) A is invertible,

(b) for all $\overrightarrow{b} \in \mathbb{R}^n$ the equation $A\overrightarrow{x} = \overrightarrow{b}$ has a solution,

(c) the columns of A span $\mathbb{R}^n$,

(d) $T_A \colon \mathbb{R}^n \to \mathbb{R}^n$, $\overrightarrow{x} \mapsto A\overrightarrow{x}$ is onto,

(e) A has a pivot position in every column,

(f) the reduced echelon form of A is I_n,

(g) $A\overrightarrow{x} = \overrightarrow{0}$ has a unique solution,

(h) the columns of A are linearly independent,

(i) T_A is one-to-one,

(j) there is an $n \times n$ matrix D with $AD = I_n$,

(k) there is an $n \times n$ matrix D with $DA = I_n$

Proof: Parts already seen in the lecture, see also textbook p. 112.