

## The invertible matrix theorem continued

The following statements are equivalent for an  $n \times n$ -matrix  $A$  to be invertible. Hence they complement 12.19:

- (l) The columns of  $A$  form a basis of  $\mathbb{R}^n$ ,
- (m)  $\text{Col}(A) = \mathbb{R}^n$
- (n)  $\text{rank } A = \dim \text{Col}(A) = n$
- (o)  $\text{Nul}(A) = \{\vec{0}\}$
- (p)  $\dim \text{Nul}(A) = 0$ .
- (r)  $\det(A) \neq 0$