TMA 4115 Matematikk 3

Lecture 10 for MBIOT5, MTKJ, MTNANO

Alexander Schmeding

NTNU

05. February 2014

Vectors

We defined **vectors** as ordered lists of numbers.

Can perform operations $+, -$ for vectors and $\cdot$ for vector and number.

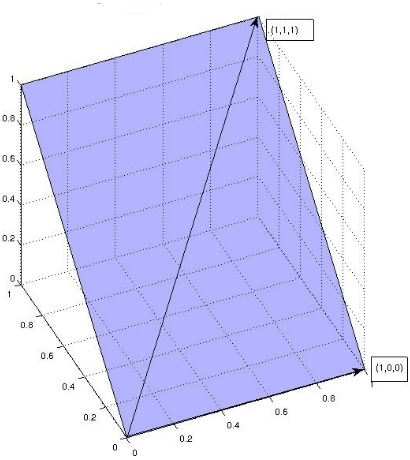
Recall that a **linear combination** of vectors $\overrightarrow{v_1}, \ldots, \overrightarrow{v_k} \in \mathbb{R}^n$ is a weighted sum

$$\sum_{l=1}^{k} r_l \overrightarrow{v_l} = r_1 \overrightarrow{v_1} + \ldots + r_k \overrightarrow{v_k}$$

For a set of vectors we introduced span $\{ \overrightarrow{v_1}, \ldots, \overrightarrow{v_k} \}$ as the **set of all linear combinations** of $\overrightarrow{v_1}, \ldots, \overrightarrow{v_k}$.

Examples of such sets in $\mathbb{R}^3$ where a point (the origin), lines through the origin and a plane (also through the origin)

Span of the vectors $(1, 1, 1)$ and $(1, 0, 0)$



The vectors are not multiples of each other (and both are not $\overrightarrow{0}$), so they span a plane in $\mathbb{R}^3$.

Linear systems vs. vector equations vs. matrices

We can always rewrite a linear system such as

$$\begin{aligned} x_1 + 5x_2 + 3x_3 + 2x_4 &= 4 \\ x_1 - 2x_3 + 2x_4 &= 0 \\ 2x_2 + 4x_3 + 2x_4 &= 1 \end{aligned}$$

as a vector equation

$$x_1 \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_2 \cdot \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix} + x_3 \cdot \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} + x_4 \cdot \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}$$

Solutions to the linear system $\leftrightarrow$ solutions to the vector equation.

Furthermore, there is a **representation** of the linear system by an augmented matrix

$$\begin{bmatrix} 1 & 5 & 3 & 2 & 4 \\ 1 & 0 & -2 & 2 & 0 \\ 0 & 2 & 4 & 2 & 1 \end{bmatrix}$$

Linear systems vs. vector equations vs. matrices II

We use the augmented matrix to solve the linear system.
However, the matrix representation is a *representation*, but not an *equation*.

Question: Can we find an equation with matrices to write the linear system (a matrix equation)?
Idea: Use the coefficients matrix of the linear system:

$$A = \begin{bmatrix} 1 & 5 & 3 & 2 \\ 1 & 0 & -2 & 2 \\ 0 & 2 & 4 & 2 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} & \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix} & \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} & \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \end{bmatrix}$$

Perhaps we can make sense of the equation $A\overrightarrow{x} = A \cdot \overrightarrow{x} = \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}$

Linear systems vs. vector equations vs. matrices III

To make sense of the equation $A\overrightarrow{x} = \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}$ for $\overrightarrow{x}$, the left hand side of the equation must coincide with the left hand side of the vector equation

$$A\overrightarrow{x} = x_1 \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_2 \cdot \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix} + x_3 \cdot \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} + x_4 \cdot \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

In fact we use this equation to define a product $A\overrightarrow{x}$ for a suitable vector $\overrightarrow{x}$.