TMA 4115 Matematikk 3

Lecture 12 for MBIOT5, MTKJ, MTNANO

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10.5 Example

Find a linearly independent subfamilie which spans the same set as

$$\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}4\\5\\6\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix} \underset{\text{augmented matrix}}{\rightsquigarrow} \begin{bmatrix}1 & 4 & 0 & 0 & 1 & 0\\2 & 5 & 1 & 1 & 1 & 0\\3 & 6 & 1 & 0 & 1 & 0\end{bmatrix}$$

$$\underset{\text{Gaussian elimination}}{\rightsquigarrow} \begin{bmatrix}1 & 1 & 0 & 0 & 1 & 0\\0 & -3 & 1 & 0 & -2 & 0\\0 & 0 & 1 & 1 & -1 & 0\end{bmatrix}$$

The vectors corresponding to basic variables form a linearly independent subfamily with the same span as before:

$$\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}4\\5\\6\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}$$

Matrix transformations

The Matrix A can be used in the equation $A\overrightarrow{x} = \overrightarrow{b}$.
Here A and $\overrightarrow{b}$ are fixed and we are searching for $\overrightarrow{x}$

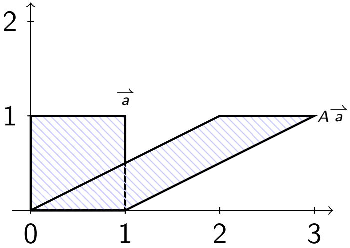
However, Matrix multiplication allows us to apply A to all possible vectors, e.g. $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ compute

$$A \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad A \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

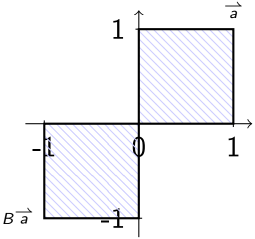
Idea: Change view of Matrices to a more dynamic concept:
Matrices give us "machines that transform vectors".

Applying matrices to 2D boxes

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \ \overrightarrow{a} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \ \overrightarrow{a} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



A yields a shear transformation



B yields a reflection

Goal for lecture: View the transformations attached to matrices as functions and study their properties!