TMA 4115 Matematikk 3

Lecture 2 for MBIOT5, MTKJ, MTNANO

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Complex numbers

A *complex number* is an expression

$$a + ib \quad \text{(may also write} \quad a + bi\text{)}$$

where a, b are real numbers and i the imaginary unit ($i^2 = -1$)

Representations of the complex number w:

$a + ib$ *normal form* (also called *standard form*), $\mathrm{Re}(w) = a$ and $\mathrm{Im}(w) = b$

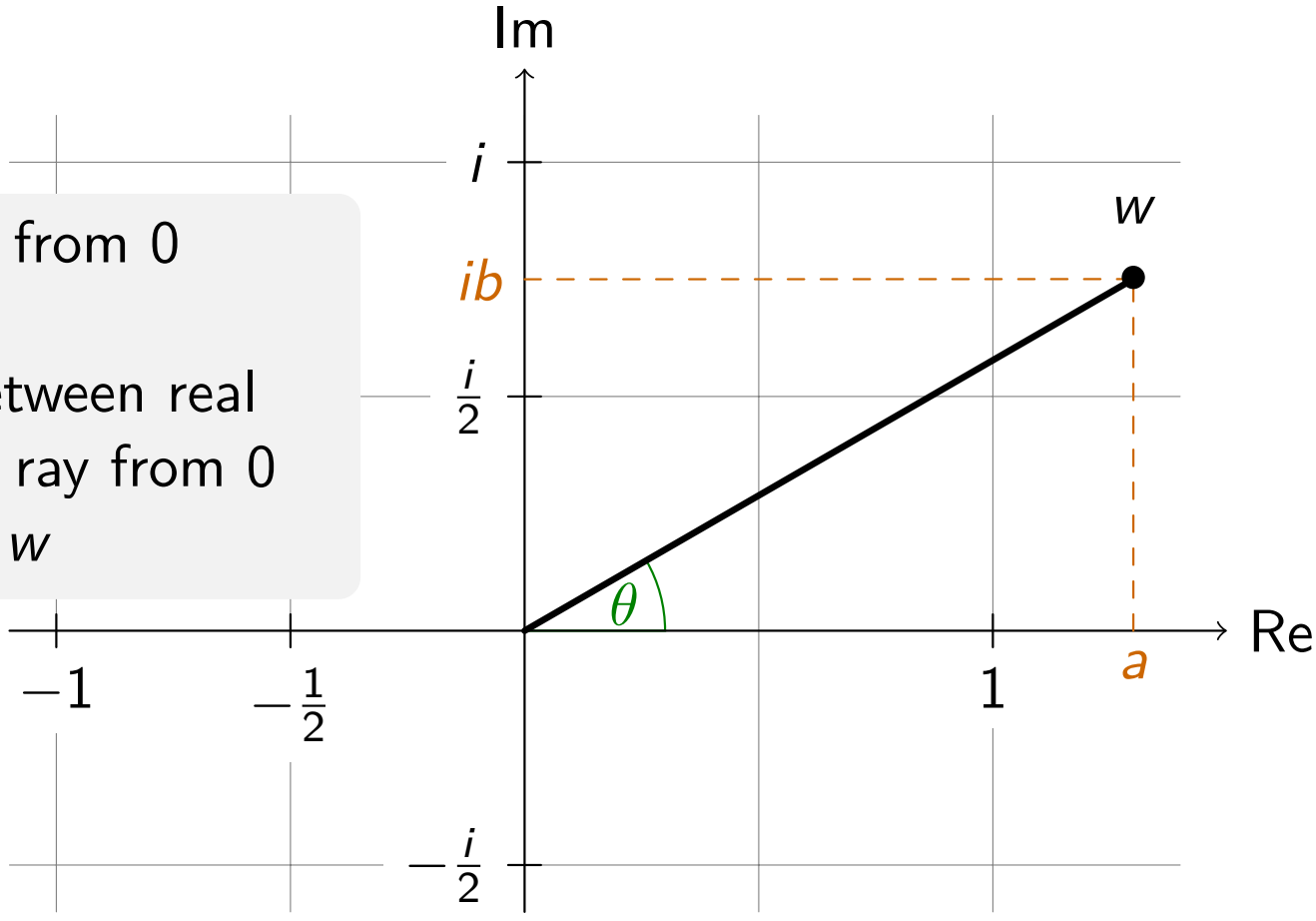
(a, b) *cartesian coordinates* for the the complex plane

(r, θ) *polar coordinates* for the complex plane and $w \neq 0$.

How do we obtain r and θ for $w = a + ib$?

r : distance from 0 to w

θ : angle between real axis and ray from 0 through w



Thus $r = |w|$ and $\theta = \text{Arg}(w)$

How to compute (r, θ) from $w = a + ib$?

We know

$$r = |w| = \sqrt{a^2 + b^2}$$

$$\tan(\theta) = \tan(\arg(a + bi)) = \frac{b}{a} \qquad \text{(if } a \neq 0\text{)}$$

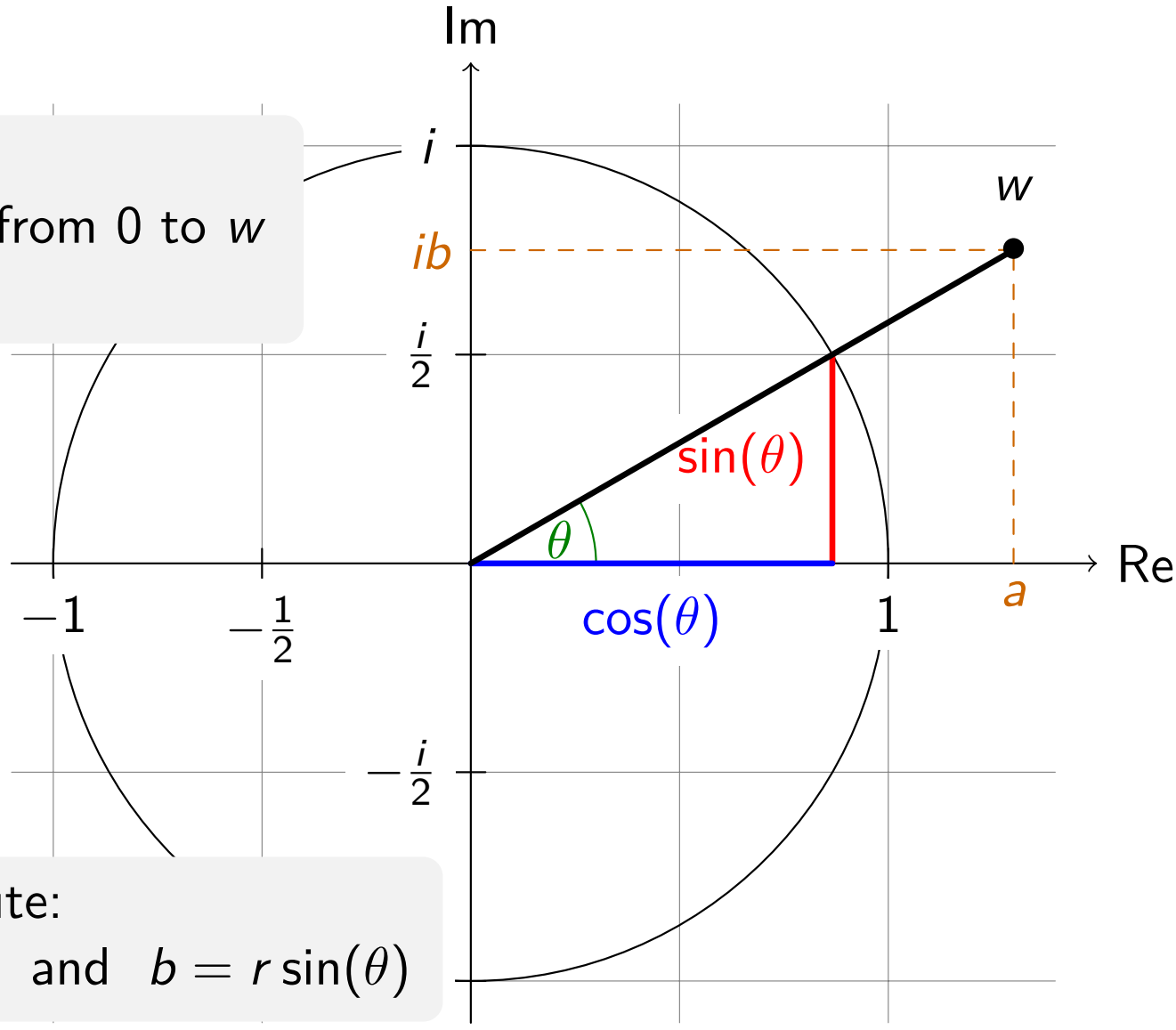
Warning: Your calculator can compute $\tan^{-1}(\frac{b}{a})$ *but*:

$$\tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{-b}{-a}\right)$$

Problem: Same number, but the angle should be different!
Solution: Use the two variable arctan function (called atan2) or use $\tan^{-1}$ and the formula for atan2 on Wikipedia
`http://en.wikipedia.org/wiki/Atan2`

Recovering coordinates from (r, θ).

Use Pythagoras Theorem and basic geometry:



We know:
r : distance from 0 to w
θ : angle

Then compute:
$a = r\cos(\theta)$ and $b = r\sin(\theta)$