TMA 4115 Matematikk 3

Lecture 3 for MBIOT5, MTKJ, MTNANO

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Complex numbers

For complex numbers we ...

- can compute sums, products, reciproals, quotients and *n*th-roots,
- use the polar form to compute products and *n*th-roots

The formula for *n*th roots in the last lecture contained an error.
Thus we repeat the formulae:

n-th roots of complex numbers

Let $z = r(\cos(\theta) + i\sin(\theta)$ be a complex number and $n \in \mathbb{N}$.

We call

$$z_1 = \sqrt[n]{r}\left(\cos\left(\frac{\theta}{n}\right) + i\sin\left(\frac{\theta}{n}\right)\right)$$

the *principal nth root of z*.

Examples:

$z = i = 1(\cos(\frac{\pi}{2}) + i\sin(\frac{\pi}{2}))$ the principal nth root
is $\cos(\frac{\pi}{2n}) + i\sin(\frac{\pi}{2n})$
$z = 0$ the principal n-th root is 0.

For $z \neq 0$ there are n distinct nth roots which can be computed via

$$
\begin{aligned}
z_1 &= \sqrt[n]{r}\left(\cos\left(\frac{\theta}{n}\right) + i\sin\left(\frac{\theta}{n}\right)\right)\\
z_2 &= \sqrt[n]{r}\left(\cos\left(\frac{\theta + 2\pi}{n}\right) + i\sin\left(\frac{\theta + 2\pi}{n}\right)\right)\\
z_3 &= \sqrt[n]{r}\left(\cos\left(\frac{\theta + 4\pi}{n}\right) + i\sin\left(\frac{\theta + 4\pi}{n}\right)\right)\\
\vdots \quad & \quad \vdots\\
z_n &= \sqrt[n]{r}\left(\cos\left(\frac{\theta + 2(n-1)\pi}{n}\right) + i\sin\left(\frac{\theta + 2(n-1)\pi}{n}\right)\right)
\end{aligned}
$$

Complex functions and the exponential map

Our next goal is to define functions depending on complex variables.

We need these functions to solve differential equations.

How do we define define a *complex function*?
- Copy the definition of a real function and use complex variables!

Real functions

Recall that a *real function* is a triple

$$f\colon U \to V, x \mapsto f(x)$$

where the parts of the triple are

$$\begin{aligned} U \subseteq \mathbb{R} &\text{ the \textit{domain} (i.e. the numbers we apply } f \text{ to)} \\ V \subseteq \mathbb{R} &\text{ the \textit{codomain} (must contain all values of } f) \\ x \mapsto f(x) &\text{ a \textit{rule} assigning to each } x \text{ an element } f(x) \end{aligned}$$

Examples:

$g\colon (0,1) \to (0,\infty), x \mapsto \frac{1}{x}$

$\sin\colon \mathbb{R} \to \mathbb{R}, x \mapsto \sin(x)$

Here (a, b) is the open interval from a to b.