TMA 4115 Matematikk 3

Lecture 8 for MBIOT5, MTKJ, MTNANO

Alexander Schmeding

NTNU

29. January 2014

7. Systems of Linear Equations

In this chapter we discuss how to solve linear equations in an efficient manner. ($\rightarrow$ Matrices, Gaussian elimination)

A **linear equation** is an equation

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n = b \tag{1}$$

with $n \in \mathbb{N}$, b and the **coefficients** $a_1, \ldots, a_n$ being real or complex numbers.

Example

$4x_1 - 5x_2 = -2$ and $x_2 = \sqrt{\pi}x_1$ are linear,
$4x_1 + x_1x_2 = 0$ and $x_2 = \sqrt{x_1}$ are not linear.

Systems of linear equations

A **system of linear equations** (or **linear system**) is a collection of one or more linear equations involving the same variables $x_1, \ldots, x_n$.

$$\textbf{Example} \quad \begin{aligned} x_1 + x_2 + 1.5x_3 &= 42 \\ x_1 - x_3 &= -7 \end{aligned}$$

A **solution** of the linear system is a list $(s_1, \ldots, s_n)$ of numbers that make each equation a true statement when we substitute each s_i for x_i, respectively.
The set of all solutions is called **solution set** of the linear system.

Two linear systems are **equivalent** if they have the same solution set.

Linear Equations in chemistry

We want to balance the reaction equation

$$\text{Ethanol} + \text{Oxygen} \longrightarrow \text{Carbondioxide} + \text{Water}$$
$$C_2H_6O + O_2 \longrightarrow CO_2 + H_2O$$

Introduce indeterminates x_1, x_2, x_3, x_4 and write

$$x_1 C_2H_6O + x_2 O_2 = x_3 CO_2 + x_4 H_2O$$

Find a solution with all $x_i \in \mathbb{Z}$. Use element relations to generate:

$$\begin{aligned} 2x_1 + 0x_2 - 1x_3 - 0x_4 &= 0 \\ 6x_1 + 0x_2 - 0x_3 - 2x_4 &= 0 \\ 1x_1 + 2x_2 - 2x_3 - 1x_4 &= 0 \end{aligned}$$

Solve these equations $\leftrightarrow$ Balance the chemical reaction

A linear equation $ax_1 + bx_2 = c$ describes a line in $\mathbb{R}^2$.
To solve simultaneously $ax_1 + bx_2 = c$ and $dx_1 + ex_2 = f$, is to find points located on both lines. Hence there may be no, one or infinitely many solutions satisfying both equations simultaneously.

We can generalize to obtain:

A system of linear equations has

- no solution, or
- exactly one solution, or
- infinitely many solutions

We call a linear system **consistent** if it has one or infinitely many solutions and **inconsistent** if it has no solution.