

TMA 4115 Matematikk 3

Lecture 8 for MBIOT5, MTKJ, MTNANO

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29. January 2014

7. Systems of Linear Equations

In this chapter we discuss how to solve linear equations in an efficient manner. (\rightarrow Matrices, Gaussian elimination)

A **linear equation** is an equation

$$a_1x_1 + a_2x_2 + \dots + a_nx_n = b \quad (1)$$

with $n \in \mathbb{N}$, b and the **coefficients** a_1, \dots, a_n being real or complex numbers.

Example

$4x_1 - 5x_2 = -2$ and $x_2 = \sqrt{\pi}x_1$ are linear,
 $4x_1 + x_1x_2 = 0$ and $x_2 = \sqrt{x_1}$ are not linear.

Systems of linear equations

A **system of linear equations** (or **linear system**) is a collection of one or more linear equations involving the same variables x_1, \dots, x_n .

Example $x_1 + x_2 + 1.5x_3 = 42$
 $x_1 - x_3 = -7$

A **solution** of the linear system is a list (s_1, \dots, s_n) of numbers that make each equation a true statement when we substitute each s_i for x_i , respectively.

The set of all solutions is called **solution set** of the linear system.

Two linear systems are **equivalent** if they have the same solution set.

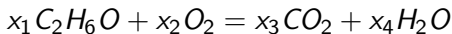
Linear Equations in chemistry

We want to balance the reaction equation

Ethanol + Oxygen \longrightarrow Carbondioxide + Water



Introduce indeterminates x_1, x_2, x_3, x_4 and write



Find a solution with all $x_i \in \mathbb{Z}$. Use element relations to generate:

$$2x_1 + 0x_2 - 1x_3 - 0x_4 = 0$$

$$6x_1 + 0x_2 - 0x_3 - 2x_4 = 0$$

$$1x_1 + 2x_2 - 2x_3 - 1x_4 = 0$$

Solve these equations \leftrightarrow Balance the chemical reaction

A linear equation $ax_1 + bx_2 = c$ describes a line in \mathbb{R}^2 .

To solve simultaneously $ax_1 + bx_2 = c$ and $dx_1 + ex_2 = f$, is to find points located on both lines. Hence there may be no, one or infinitely many solutions satisfying both equations simultaneously.

We can generalize to obtain:

A system of linear equations has

- no solution, or
- exactly one solution, or
- infinitely many solutions

We call a linear system **consistent** if it has one or infinitely many solutions and **inconsistent** if it has no solution.