

n -th roots of complex numbers

Let $z = r(\cos(\theta) + i \sin(\theta))$ be a complex number and $n \in \mathbb{N}$.

We call

$$z_1 = \sqrt[n]{r} \left(\cos \left(\frac{\theta}{n} \right) + i \sin \left(\frac{\theta}{n} \right) \right)$$

the *principal n th root* of z .

Examples:

$z = i = 1(\cos(\frac{\pi}{2}) + i \sin(\frac{\pi}{2}))$ the principal n th root
is $\cos(\frac{\pi}{2n}) + i(\frac{\pi}{2n})$

$z = 0$ the principal n -th root is 0.

For $z \neq 0$ there are n distinct n th roots which can be computed via

$$z_1 = \sqrt[n]{r} \left(\cos \left(\frac{\theta}{n} \right) + i \sin \left(\frac{\theta}{n} \right) \right)$$

$$z_2 = \sqrt[n]{r} \left(\cos \left(\frac{\theta + 2\pi}{n} \right) + i \sin \left(\frac{\theta + 2\pi}{n} \right) \right)$$

$$z_3 = \sqrt[n]{r} \left(\cos \left(\frac{\theta + 4\pi}{n} \right) + i \sin \left(\frac{\theta + 4\pi}{n} \right) \right)$$

$$\vdots \quad \vdots$$

$$z_n = \sqrt[n]{r} \left(\cos \left(\frac{\theta + 2(n-1)\pi}{n} \right) + i \sin \left(\frac{\theta + 2(n-1)\pi}{n} \right) \right)$$