The adjugate matrix

Let A be an $n \times n$ square matrix and A_{ij} be the matrix obtained from A by omitting the ith row and the jth column. Then we define the **adjugate** of A:

$$\begin{aligned}
\text{adj } A &= \left[(-1)^{i+j}|A_{ij}|\right]^T \\
&= \begin{bmatrix} (-1)^{1+1}|A_{ij}| & (-1)^{1+2}|A_{12}| & \cdots & (-1)^{1+n}|A_{in}| \\ (-1)^{2+1}|A_{21}| & (-1)^{2+2}|A_{22}| & \cdots & (-1)^{2+n}|A_{2n}| \\ \vdots & \vdots & \vdots & \vdots \\ (-1)^{n+1}|A_{n1}| & (-1)^{n+2}|A_{n2}| & \cdots & (-1)^{n+n}|A_{nn}| \end{bmatrix}^T
\end{aligned}$$

If A is invertible then $A^{-1} = \frac{1}{\det A} \text{adj } A$.

For the matrix $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ one computes adj $B = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$