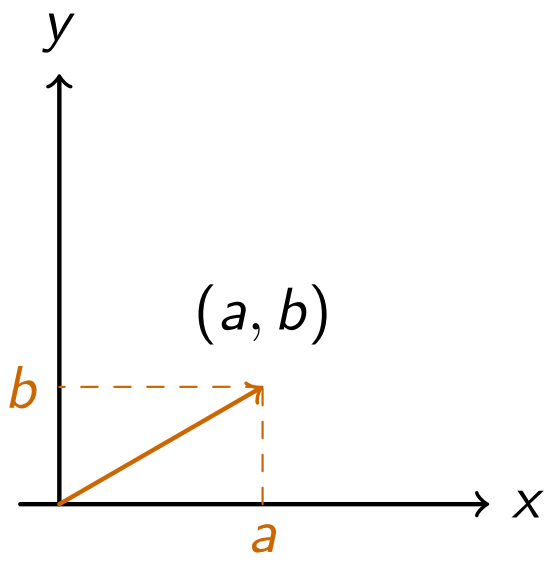
In today's lecture we will

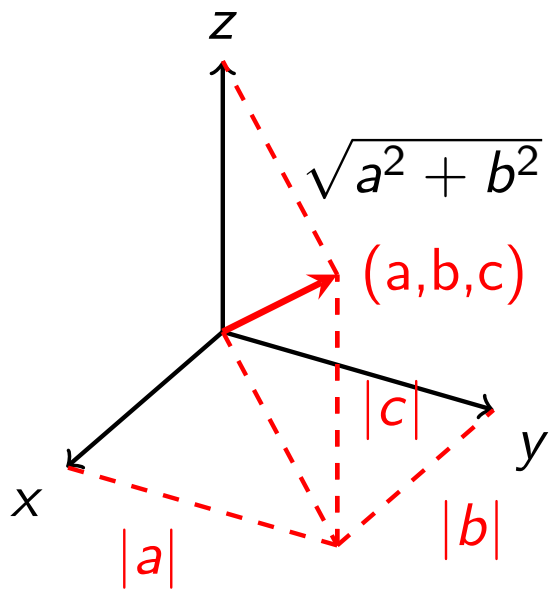
- explore the geometry of $\mathbb{R}^n$.
- define the length of vectors
- investigate when vectors are orthogonal

Geometry of $\mathbb{R}^n$: Length of vectors

Length in 2d:



Length in 3d:



Pythagoras:

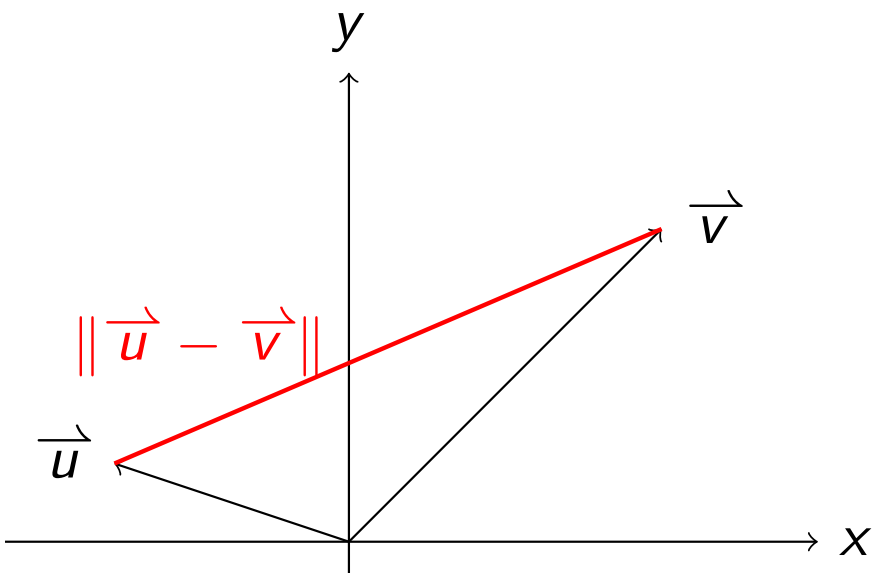
length of $\begin{bmatrix} a \\ b \end{bmatrix}$ is $\sqrt{a^2+b^2}$

Pythagoras (2 times!):

length of $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ is $\sqrt{a^2+b^2+c^2}$

Distance between two vectors

How can we measure the distance between points?



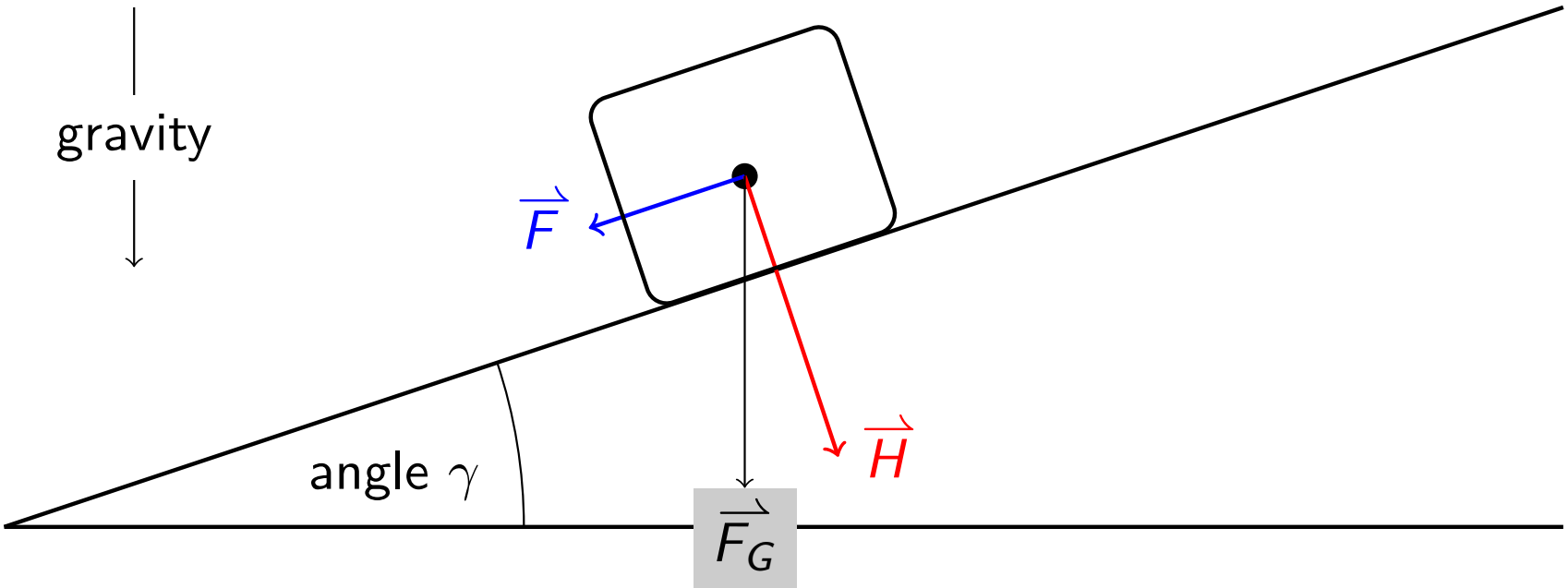
Distance between vectors

$\text{dist}(\vec{u}, \vec{v}) = \|\vec{u} - \vec{v}\|$

Note: $\|\vec{u} - \vec{v}\| = \|\vec{v} - \vec{u}\|$.

Example: Splitting forces in physics

Say we know the weight of a block on a slope:



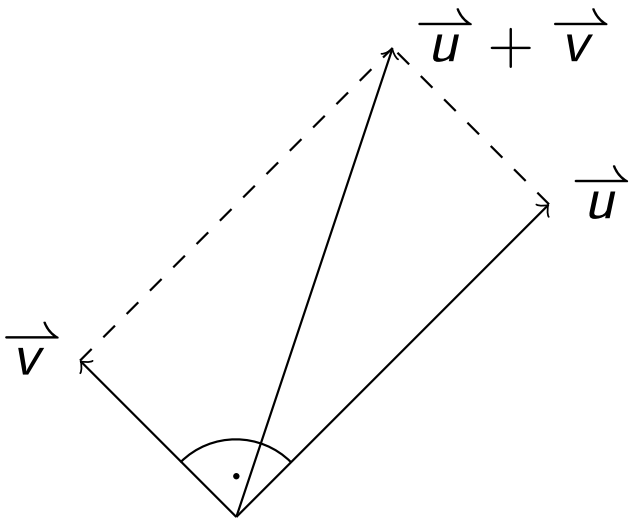
Can compute $\overrightarrow{F}_G$ from the weight but we want: $\overrightarrow{F}$, the force acting on the block in the direction of the slope.

Note: $\overrightarrow{F}$ and $\overrightarrow{H}$ are orthogonal!

$\rightarrow$ Idea: split $\overrightarrow{F}_G$ in orthogonal components

What does orthogonal mean?

If vectors $\vec{u}$ and $\vec{v}$ in $\mathbb{R}^2$ meet in a right angle, they are **perpendicular** (or **orthogonal**):



Pythagoras theorem

$\|\vec{u} + \vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2$ Comparing both sides, the equation holds if and only if:

$$\vec{u} \cdot \vec{v} = 0$$

Idea: Use this to define orthogonal vectors in general settings.