TMA 4115 Matematikk 3

Lecture 13 for MTFYMA

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- continue example how linearly independent subfamilies are obtained,
- dynamic view of matrices (matrix transformations)

A family of vectors $\overrightarrow{v}_1, \ldots, \overrightarrow{v}_k$ are called

- linearly independent if

$$\sum_{i=1}^{n} r_i \overrightarrow{v}_i = \overrightarrow{0} \tag{1}$$

  only admits the trivial solution $r_1 = r_2 = \ldots = r_n = 0$.

- If (1) admits a non trivial solution (at least one $r_i \neq 0$) then the vectors are linearly dependent (call (1) linear dependency relation).

Test for linear independence

Use Gaussian elimination on the matrix $[\overrightarrow{v}_1 \ \overrightarrow{v}_2 \ldots \overrightarrow{v}_k]$. The vectors are linearly independent if and only if every column is a pivot column.

Example (How to generate a linearly independent subfamily)

Find a linearly independent subfamilie with the same span as

$$\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}4\\5\\6\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}$$

linearly independent subfamily: $\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}4\\5\\6\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}$

$$\underset{\text{coefficient matrix}}{\rightsquigarrow} \begin{bmatrix}1 & 4 & 0 & 0 & 1\\2 & 5 & 1 & 1 & 1\\3 & 6 & 1 & 0 & 1\end{bmatrix} \rightsquigarrow \begin{bmatrix}1 & 4 & 0 & 0 & 1\\0 & -3 & 1 & 1 & -1\\0 & -6 & 1 & 0 & -2\end{bmatrix}$$

$$\underset{\text{Gaussian elimination}}{\rightsquigarrow} \begin{bmatrix}1 & 1 & 0 & 0 & 1\\0 & -3 & 1 & 0 & -2\\0 & 0 & 1 & 1 & -1\end{bmatrix}$$

The vectors corresponding to basic variables form a linearly independent subfamily (with the same span)

Matrix transformations

In the equation $A\overrightarrow{x} = \overrightarrow{b}$, A and $\overrightarrow{b}$ are fixed and we search for $\overrightarrow{x}$

However, Matrix multiplication allows us to apply A to arbitrary vectors, e.g. $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ compute

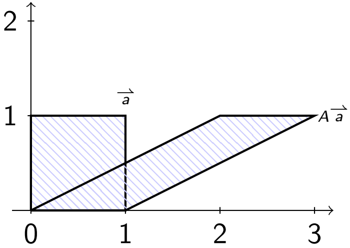
$$A \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad A \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

Change view of Matrices to a more dynamic concept:

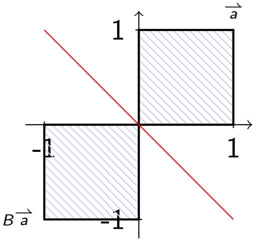
Matrices as “machines that transform vectors”.

Applying matrices to 2D boxes

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \ \vec{a} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \ \vec{a} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



A: shear transformation



B: reflection (along red line)

Goal for lecture: View the transformations attached to matrices as functions and study their properties!