# TMA 4115 Matematikk 3

## Lecture 2 for MTFYMA

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# Complex numbers

A *complex number* is an expression

$$a + ib \quad \text{(may also write} \quad a + bi)$$

where $a, b$ are real numbers and $i$ the imaginary unit ($i^2 = -1$)

**Representations of a complex number** $w$:

$a + ib$ *normal form* (or *standard form*), $\mathrm{Re}(w) = a$ and $\mathrm{Im}(w) = b$

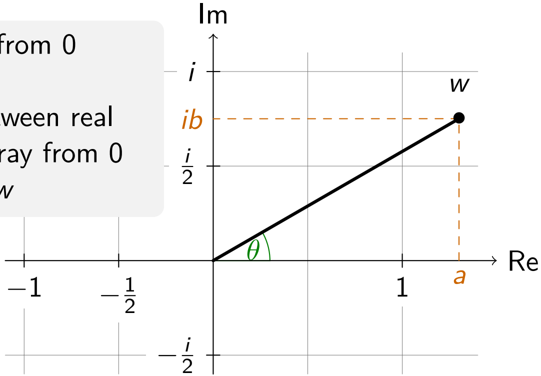
$(a, b)$ *cartesian coordinates* for the complex plane

$(r, \theta)$ *polar coordinates* for the complex plane and $w \neq 0$.

# How do we obtain $r$ and $\theta$ for $w = a + ib$?

$r$ : distance from 0 to $w$

$\theta$ : angle between real axis and ray from 0 through $w$




$r = |w|$ and $\theta \in \arg(w) = \{\dots, \theta - 2\pi, \theta, \theta + 2\pi, \theta + 4\pi, \dots\}$

Recall if $-\pi < \theta \leq \pi$ then $\theta = \operatorname{Arg}(w)$ "principal argument".

# How to compute $(r, \theta)$ from $w = a + ib$?

We know

$$r = |w| = \sqrt{a^2 + b^2}$$

$$\tan(\theta) = \tan(\arg(a + bi)) = \frac{b}{a} \qquad \text{(if } a \neq 0\text{)}$$

**Warning**: Your calculator can compute $\tan^{-1}(\frac{b}{a})$ *but*:

$$\tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{-b}{-a}\right)$$

**Problem**: Same number, but the angle should be different!

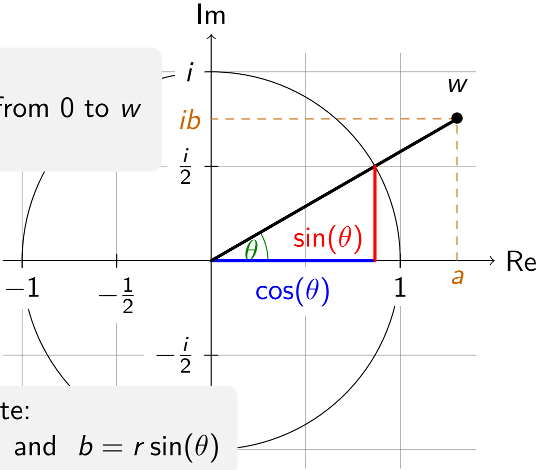
**Solution**:

Use the two variable arctan function (called atan2)
or use $\tan^{-1}$ and the formula for atan2 on Wikipedia
`http://en.wikipedia.org/wiki/Atan2`

# Recovering coordinates from $(r, \theta)$.

Use Pythagoras Theorem and basic geometry:

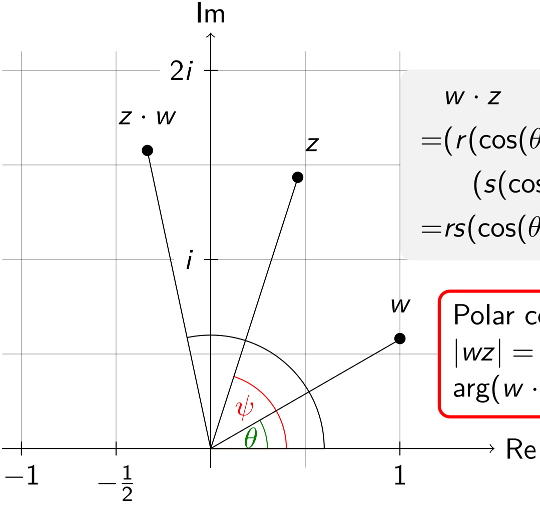
We know:
$r$ : distance from 0 to $w$
$\theta$ : angle




Then compute:
$a = r\cos(\theta)$ and $b = r\sin(\theta)$

# Multiplying complex numbers graphically.

Given complex numbers $w = (r, \theta)$ and $z = (s, \psi)$ what is $w \cdot z$?




$$
\begin{aligned}
& w \cdot z \\
=& (r(\cos(\theta) + i\sin(\theta)))\cdot \\
& \quad (s(\cos(\psi) + i\sin(\psi)) \\
=& rs(\cos(\theta+\psi) + i\sin(\theta+\psi)
\end{aligned}
$$

Polar coordinates of $w \cdot z$:
$|wz| = |w||z|$ and
$\arg(w \cdot z) = \arg(w) + \arg(z)$