TMA 4115 Matematikk 3

Lecture 9 for MTFYMA

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In this lecture we discuss:

- Systems of linear equations...
- ... their connection to matrices.
- Gaussian elimination
- solution sets to linear equations

Looking back and beyond

In the last chapters we wanted to construct solutions for

$$y'' + p(t)y' + q(t)y = f(t)$$

Recall that...

- values of parameters in solutions are determined by solving linear equations.
- the Wronskian determines linear independence of solutions
- ...and in **6.1** the Wronskian showed that the system of linear equations is solvable.

Now: Study these objects to deal with linear equations efficiently.

7. Systems of Linear Equations

In this chapter we discuss how to solve linear equations in an efficient manner. ($\rightarrow$ Matrices, Gaussian elimination)

A **linear equation** is an equation

$$a_1 x_1 + a_2 x_2 + \ldots + a_n x_n = b \tag{1}$$

with $n \in \mathbb{N}$, b and the **coefficients** $a_1, \ldots, a_n$ being real or complex numbers.

Example

$4x_1 - 5x_2 = -2$ and $x_2 = \sqrt{\pi} x_1$ are linear,
$4x_1 + x_1 x_2 = 0$ and $x_2 = \sqrt{x_1}$ are not linear.

Systems of linear equations

A **system of linear equations** (or **linear system**) is a collection of linear equations involving the same variables $x_1, \ldots, x_n$.

A **solution** of the linear system is a list $(s_1, \ldots, s_n)$ of numbers such that each equation is true statement when we replace each x_i with s_i, respectively.
The set of all solutions is called **solution set** of the linear system.

Two linear systems are **equivalent** if they have the same solution set.

Linear Equations in chemistry

We want to balance the reaction equation

$$\text{Ethanol} + \text{Oxygen} \longrightarrow \text{Carbondioxide} + \text{Water}$$
$$C_2H_6O + O_2 \longrightarrow CO_2 + H_2O$$

Introduce indeterminates x_1, x_2, x_3, x_4 and write

$$x_1 C_2H_6O + x_2 O_2 = x_3 CO_2 + x_4 H_2O$$

Note: $x_1 C_2H_6O$ has $2x_1$ atoms of carbon, $6x_1$ atoms of hydrogen and x_1 atoms of oxygen.

Idea

To generate a system of linear equations we generate an equation for each type of atom in the equation.

Linear Equations in chemistry

$$x_1 C_2 H_6 O + x_2 O_2 = x_3 CO_2 + x_4 H_2 O$$

Find a solution with all $x_i \in \mathbb{Z}$. Use element relations to generate:

$$\begin{aligned} 2x_1 + 0x_2 - 1x_3 - 0x_4 &= 0 \\ 6x_1 + 0x_2 - 0x_3 - 2x_4 &= 0 \\ 1x_1 + 2x_2 - 2x_3 - 1x_4 &= 0 \end{aligned}$$

Solve these equations $\leftrightarrow$ Balance the chemical reaction

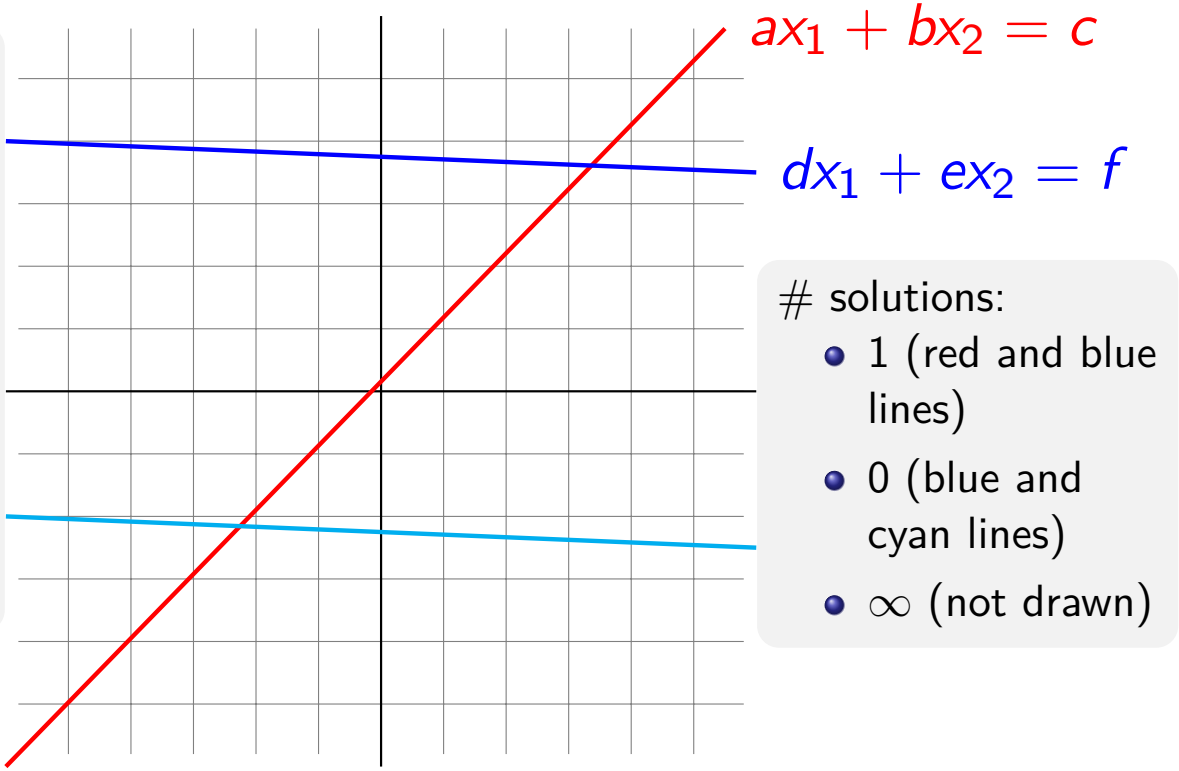
Solution sets of linear equations

Linear equations (of two variables) describe lines in the plane.

Solutions of

$$ax_1 + bx_2 = c$$
$$dx_1 + ex_2 = f$$

are points which simultaneously lie on both lines.



solutions:

- 1 (red and blue lines)
- 0 (blue and cyan lines)
- ∞ (not drawn)

Similar arguments apply to systems with more variables.

Solution sets of linear equations

A system of linear equations has

- no solution, or
- exactly one solution, or
- infinitely many solutions

We call a linear system **consistent** if it has one or infinitely many solutions and **inconsistent** if it has no solution.