TMA4115 Calculus 3
Spring 2016

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Norges teknisk–naturvitenskapelige universitet
Institutt for matematiske fag

1.3.1. Let $z_1 = 2 - i$ and $z_2 = 1 + i$. Use the parallelogram law to construct each of the following vectors.

(a) $z_1 + z_2$ (b) $z_1 - z_2$ (c) $2z_1 - 3z_2$

1.3.5. Find the following.

(a) $\left|\dfrac{1+2i}{-2-i}\right|$ (b) $|(\overline{1+i})(2-3i)(4i-3)|$ (c) $\left|\dfrac{i(2+i)^3}{(1-i)^2}\right|$

(d) $\left|\dfrac{(\pi+i)^{100}}{(\pi-i)^{100}}\right|$

1.3.7. Find the argument of each of the following complex numbers and write each in polar form.

(a) $-\dfrac{1}{2}$ (b) $-3+3i$ (c) $-\pi i$ (d) $-2\sqrt{3}-2i$

(e) $(1-i)(-\sqrt{3}+i)$ (f) $(\sqrt{3}-i)^2$ (g) $\dfrac{-1+\sqrt{3}i}{2+2i}$

(h) $\dfrac{-\sqrt{7}(1+i)}{\sqrt{3}+i}$

1.3.13. Decide which of the following statements are always true.

(a) $\text{Arg } z_1 z_2 = \text{Arg } z_1 + \text{Arg } z_2$, if $z_1 \neq 0$, $z_2 \neq 0$.
(b) $\text{Arg } \bar{z} = -\text{Arg } z$, if z is not a real number.
(c) $\text{Arg } (z_1/z_2) = \text{Arg } z_1 - \text{Arg } z_2$, if $z_1 \neq 0$, $z_2 \neq 0$.
(d) $\arg z = \text{Arg } z + 2\pi k$, for $k = 0, \pm 1, \pm 2, \ldots$, if $z \neq 0$.

1.4.1. Write each of the given numbers in the form $a + bi$.

(a) $e^{-i\pi/4}$ (b) $\dfrac{e^{1+3i\pi}}{e^{-1+i\pi/2}}$ (c) e^{e^i}

1.4.3. Write each of the given numbers in the polar form $re^{i\theta}$.

(a) $\dfrac{1-i}{3}$ (b) $-8\pi(1+\sqrt{3}i)$ (c) $(1+i)^6$

1.4.7. Show that $e^z = e^{z+2\pi i}$ for all z. (*The exponential function is periodic with period* $2\pi i$.)

1.5.5. Find all the values of the following.

(a) $(-16)^{1/4}$ (b) $1^{1/5}$ (c) $i^{1/4}$

(d) $(1-\sqrt{3}i)^{1/3}$ (e) $(i-1)^{1/2}$ (f) $\left(\frac{2i}{1+i}\right)^{1/6}$

1.5.7. Solve the following equation.

(c) $z^2 - 2z + i = 0$

1.5.9. Solve the equation $z^3 - 3z^2 + 6z - 4 = 0$.