Norges teknisk–naturvitenskapelige universitet
Institutt for matematiske fag

TMA4115 Calculus 3
Spring 2016

Øving 4

4.5 Inhomogeneous Equations; the Method of Undetermined Coefficients

In Exercises 1 and 2, use the technique demonstrated in Example 5.6 to find a particular solution for the given differential equation.

1. $y'' + 3y' + 2y = 4e^{-3t}$ 2. $y'' + 6y' + 8y = -3e^{-t}$

In Exercise 5, use the form $y_p = a\cos\omega t + b\sin\omega t$, as in Example 5.8, to help find a particular solution for the given differential equation.

5. $y'' + 4y = \cos 3t$

In Exercises 10 and 11, use the complex method , as in Example 5.12, to find a particular solution for the differential equation.

10. $y'' + 4y = \cos 3t$ 11. $y'' + 9y = \sin 2t$

30. If $y_f(t)$ us a solution of

$$y'' + py' + qy = f(t)$$

and $y_g(t)$ is a solution of

$$y'' + py' + qy = g(t)$$

show that $z(t) = \alpha y_f(t) + \beta y_g(t)$ is a solution of

$$y'' + py' + qy = \alpha f(t) + \beta g(t)$$

Use the technique suggested by Examples 5.23 and 5.26, as well as Exercise 30, to help find particular solutions for the differential equations in Exercises 36 and 37.

36. $y'' + 2y' + 2y = 3\cos t - \sin t$ 37. $y'' + 4y' + 4y = e^{-2t} + \sin 2t$

4.6 Variation of Parameters

For Exercises 1,4, and 5, find a particular solution to the given second-order differential equation.

1. $y'' + 9y = \tan 3t$ 4. $x'' - 2x' - 3x = 4e^{3t}$ 5. $y'' - 2y' + y = e^t$

13. Verify that $y_1(t) = t$ and $y_2(t) = t^{-3}$ are solutions to the homogeneous equation

$$t^2y''(t) + 3ty'(t) - 3y(t) = 0.$$

Use variation of parameters to find the general solution to

$$t^2y''(t) + 3ty'(t) - 3y(t) = \frac{1}{t}.$$

4.7 Forces Harmonic Motion

1. In the narrative (Case 1), the substitution $x_p = a\cos\omega t + b\sin\omega t$ produced

$$x_p = \frac{A}{\omega_0^2 - \omega^2}\cos\omega t$$

as a particular solution of $x'' + \omega_0^2 x = A\cos\omega t$, when $\omega \neq \omega_0$.

(a) Use the substitution $x_p = a\cos\omega t$ to produce the same result.

(b) Use the substitution $x_p = ae^{i\omega t}$ to produce the same result.

9. A 1-kg mass is attached to a spring ($k = 4kg/s^2$) and the system is allowed to come to rest. The spring-mass system is attached to a machine that supplies an external driving force $f(t) = 4\cos\omega t$ Newtons. The system is started from equilibrium, the mass having no initial displacement nor velocity. Ignore any damping forces.

(a) Find the position of the mass as a function of time.

(b) Place your answer in the form $x(t) = A\sin\delta t\sin\bar{\omega}t$. Select an ω near the natural frequency of the system to demonstrate the "beating" of the system. Sketch a plot that shows the "beats" and include the envelope of the beating motion in your plot (see Exercise 2).

45. A 50-g mass stretches a spring $10cm$. As the system moves through the air, a resistive force is supplied that is proportional to, but opposite the velocity, with magnitude $0.01v$. The system is hooked to a machine that applies a driving force to the mass that is equal to $F(t) = 5\cos 4.4t$ Newtons. If the system is started from equilibrium (no displacement, no velocity), find the position of the mass as a function of time. *Hint:* Remember that $1000g = 1kg$ and $100cm = 1m$.