TMA4115 Calculus 3
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1.1 System of Linear Equations

Solve the systems in Exercise **13** and **14**.

13.

$$\begin{aligned} x_1 - 3x_3 &= 8 \\ 2x_1 + 2x_2 + 9x_3 &= 7 \\ x_2 + 5x_3 &= -2 \end{aligned}$$

14.

$$\begin{aligned} x_1 - 3x_2 &= 5 \\ -x_1 + x_2 + 5x_3 &= 2 \\ x_2 + x_3 &= 0 \end{aligned}$$

25. (Optional Extra) Find an equation involving g, h, and k that makes this augmented matrix correspond to a consistent system:

$$\begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{bmatrix}$$

1.2 Row Reduction and Echelon Forms

3. Row reduce the matrix to reduced echelon form. Circle the pivot positions in the final matrix and in the original matrix, and list the pivot columns.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$$

Find the general solutions of the system whose augmented matrices are given in Exercise **7** and **11**.

7.

$$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{bmatrix}$$

11.

$$\begin{bmatrix} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{bmatrix}$$

17. Determine the value(s) of h such that the matrix is the augmented matrix of a consistent linear system.

$$\begin{bmatrix} 2 & 3 & h \\ 4 & 6 & 7 \end{bmatrix}$$

29. (Optional Extra) A system of linear equations with fewer equation the unknowns is sometimes called an *underdetermined system*. Suppose that such a system happens to be consistent. Explain why there must be infinite number of solutions.

1.3 Vector Equations

5. Write a system of equations that is equivalent to the given vector equations.

$$x_1 \begin{bmatrix} 6 \\ -1 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \\ -5 \end{bmatrix}$$

11. Determine if $\mathbf{b}$ is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$.

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$$

17. Let $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ h \end{bmatrix}$. For what value(s) of h is $\mathbf{b}$ in the plane spanned by $\mathbf{a}_1$ and $\mathbf{a}_2$?

22. Construct a 3×3 matrix A, with nonzero entries, and a vector $\mathbf{b} \in \mathbb{R}^3$ such that $\mathbf{b}$ is *not* in the set spanned by the columns of A.

1.4 The Matrix Equation $\mathbf{Ax} = \mathbf{b}$

1. Compute the products using (a) the definition, as in Example 1, and (b) the row-vector rule for computing $A\mathbf{x}$. If a product is undefined, explain why.

$$\begin{bmatrix} -4 & 2 \\ 1 & 6 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 7 \end{bmatrix}$$

8. Use the definition of $A\mathbf{x}$ to write the vector equation as a matrix equation.

$$z_1 \begin{bmatrix} 4 \\ -2 \end{bmatrix} + z_2 \begin{bmatrix} -4 \\ 5 \end{bmatrix} + z_3 \begin{bmatrix} -5 \\ 4 \end{bmatrix} + z_4 \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 13 \end{bmatrix}$$

10. Write the system first as a vector equation and then as a matrix equation.

$$\begin{aligned} 8x_1 - x_2 &= 4 \\ 5x_1 + 4x_2 &= 1 \\ x_1 - 3x_2 &= 2 \end{aligned}$$

17. How many rows of A contain a pivot position? Does the equation $A\mathbf{x} = \mathbf{b}$ have a solution for each $\mathbf{b}$ in $\mathbb{R}^4$?

$$A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix}$$

32. (Optional Extra) Could a set of three vectors in $\mathbb{R}^4$ span all of $\mathbb{R}^4$? Explain. What about n vectors in $\mathbb{R}^m$ when n is less than m?

35. (Optional Extra) Let A be a 3×4 matrix, let $\mathbf{y}_1$ and $\mathbf{y}_2$ be vectors in $\mathbb{R}^3$, and let $\mathbf{w} = \mathbf{y}_1 + \mathbf{y}_2$. Suppose $\mathbf{y}_1 = A\mathbf{x}_1$ and $\mathbf{y}_2 = A\mathbf{x}_2$ for some vectors $\mathbf{x}_1$ and $\mathbf{x}_2$ in $\mathbb{R}^4$. What fact allows you to conclude that the system $A\mathbf{x} = \mathbf{w}$ is consistent? (*Note:* $\mathbf{x}_1$ and $\mathbf{x}_2$ denote vectors, not scalar entries in vectors).